Materials for the course
Modelling Financial Risk with R

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Contents

About

This script contains materials for the course

Modelling financial risk with `R`.

The course also contains R codes that can be found on the page:

`http://web.sgh.waw.pl/~mrubas/`

As additional materials we recommend:

- Jon Danielsson 2011. "Financial Risk Forecasting", Wiley
  `https://www.financialriskforecasting.com/`
- Alexander C., 2009. "Market Risk Analysis", Wiley

Topic 1

Introduction

- Course requirements
- Additional material
- `R` package

Aims

Block 1

1. Discussing financial series characteristics
2. Presenting financial time series models
3. Prezentacja metod liczenia VaR

Block 2

1. Backtesting
2. Stress tests

Additionally

1. Programming in R
2. Developing presentation and public speech skills

Materials

Main materials:
- Script
- R codes

Available at course page:
web.sgh.waw.pl/~mrubas

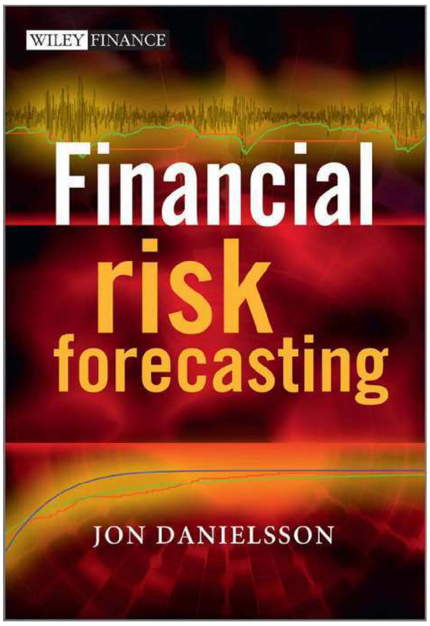
Recommended books:

Danielsson J. 2011. Financial Risk Forecasting, Wiley

Dowd K., 2005. Measuring Market Risk, Wiley
Alexander C., 2009. Market Risk Analysis, Wiley
Jorion P., 2007. Value at risk. McGraw-Hill

Internet resources:

RiskMetrics – technical document: link



Meetings outline

Block 1

i. Introduction to R
ii. Time series in R (zoo, Quandl, apply, ggplot2)
iii. Financial time series characteristics
iv. VaR & ES: unconditional distribution models
v. VaR & ES: volatility clustering (EWMA and GARCH)
vi. Presentations

Block 2

i. VaR & ES for longer horizons
ii. Backtesting
iii. Stress tests
iv. Presentations

Grades

Points are attributed for:

- 20 points for 2 presentations
- 10 points for the exam
- 2 points for activity

points	≤ 15	≤ 18	≤ 21	≤ 24	≤ 27	>27
grade	2.0	3.0	3.5	4.0	4.5	5.0

What is R

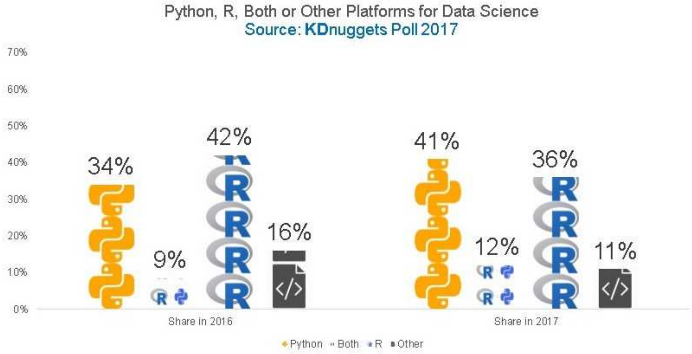
- Environment for statistical calculations and visualization of results, created by Robert Gentleman and Ross Ihaka at the University of Auckland in 1996. The name comes from the first letters of the authors' names and is a reference to the S language
- GNU R is distributed as source code and in binary form with many distributions for Linux, Microsoft Windows and Mac OS
- R is used in many well-known companies, including Facebook, Google, Merck, Altera, Pfizer, LinkedIn, Shell, Novartis, Ford, Mozilla and Twitter.
- Producers of commercial statistical packages (SPSS, SAS, Statistica) offer dedicated mechanisms ensuring their cooperation with R
- R provides a wide range of statistical techniques (linear and nonlinear modeling, classical statistical tests, time series analysis, classification, clustering, ...) and graphical.
- In addition, R is extendable with additional packages and user-written scripts.

* On the basis of information from Wikipedia

Why R

1. **Popularity**

R is also the name of a popular programming language used by a growing number of data analysts inside corporations and academia



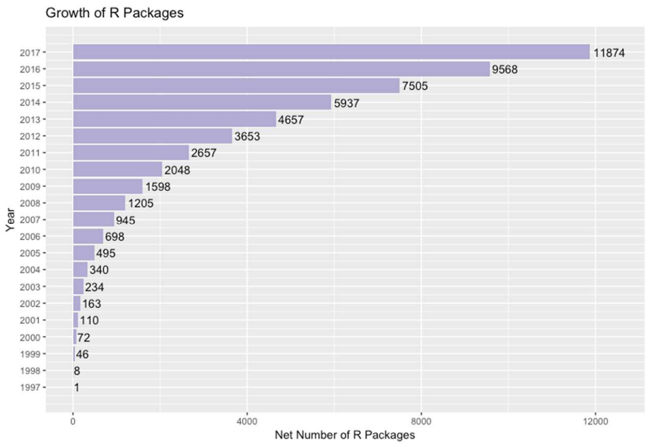
Dlaczego R

2. Comprehensiveness

„The great beauty of R is that you can modify it to do all sorts of things," said Hal Varian, chief economist at Google. "And you have a lot of prepackaged stuff that's already available, so you're standing on the shoulders of giants."

3. Price

„R first appeared in 1996, when the statistics professors Ross Ihaka and Robert Gentleman of the University of Auckland in New Zealand released the code as a **free software package**."



R – links

Webpage of R project

https://www.r-project.org/

Materials:

P. Kuhnert & B. Venables, An Introduction to R: Software for Statistical Modeling & Computing

P. Biecek, Przewodnik po pakiecie R

Rproject, **An Introduction to R**

Topic 1: exercises

Exercise 1.1.

1. Download and unzip to folder Rcodes/funds investment funds prices from bossa.pl
   - http://bossa.pl/pub/fundinwest/mstock/mstfun.zip
   - http://bossa.pl/pub/ofe/mstock/mstfun.lst

2. Select 2-3 funds of different characteristics with price history of at least 5 years

3. Analyze the profile of these funds using Key Investor Information Document - KIID (kluczowe informacje dla inwestorów)

Topic 2

Financial times series

- Downloading financial series to `R`
- `zoo` package in `R`
- Simple and logarithmic rate of return
- Moments of returns distribution
- Financial series characteristics
- QQ plot
- t-Student distribution

Importing financial time series

Quandl package

```
> require(Quandl)
> cpiUS <-  Quandl("FRED/CPIAUCNS", type = "zoo")   ## CPI USA
> brent <-  Quandl("EIA/PET_RBRTE_M", type = "zoo") ##ceny ropy brent
```

Quantmod: Yahoo, Google, Oanda,

```
> require(quantmod)
> getSymbols("SPY", src = "yahoo")
```

Importing from local files: csv, xls, xlsx, xml,…

Interaction with popular databases: MySQL, PostgreSQL, MS SQL Server,…

Time series in R, zoo

- Time Series – TS – is a series of values $X_{t_1}, X_{t_2}, X_{t_3}, \dots$; where $t_1 < t_2 < t_3 < \cdots$ are ordered time indices.
- R packages to work with TS: tseries, timeSeries, tis, stats, **zoo**, xts, …
- zoo objects consist of coredata (vector or matrix) and time index:

```
>  ts.zoo <- zoo(values , order.by = timeIdx)
## values  – numeric or matrix class
## timeIdx – Date class, also yearmon, yearqtr, POSICct, timeDate
>  index(ts.zoo)    ## time index
>  coredata(ts.zoo) ## vector or matrix
>  index(ts.zoo)    <- newTimeIdx
>  coredata(ts.zoo) <- newValues
```

- zoo objects helpful to work with time windows, merging series or frequency conversion (daily → weekly → monthly,…)

Dates

Date object represents daily data as the numer of days from 01-01-1970

```
> mydate <- as.Date("01-01-1970", format = "%d-%m-%Y")
> weekdays(mydate)   ##months(mydate)   quarters(mydate)
> mydate + 1
> mydate <- mydate – 5
```

difftime objects

```
> mydate1 <- as.Date("01-11-1990", format = "%d-%m-%Y")
> mydate – mydate1
```

Sequence of dates

```
> seq(from=mydate, to=mydate1, by="5 months")
> seq(from=mydate, by="2 months", length.out=20)
```

lubridate package helps to work with dates

```
> dmy("01-01-1970") + years(2)
> dmy("01-01-1970") + (0:19)*months(2)
> wday(mydate)
```

zoo objects

Merging objects

```
> merge(ts.zoo.1, ts.zoo.2)            ## full merge
> merge(ts.zoo.1, ts.zoo.2, all=FALSE) ## inner merge
```

Windows

```
> window(ts.zoo, start=as.Date("2007-01-05"), end=as.Date("2008-02-01"))
```

Lags and leads

```
> lag(ts.zoo, -1) ## previous value
> lag(ts.zoo, 1)  ## next value
```

Differences

```
>diff(ts.zoo)
```

Rates of returns

```
> diff(ts.zoo)/lag(ts.zoo, -1)  ## simle
> diff(log(ts.zoo))             ## log-returns
```

Loops with *apply* function

Rolling std. deviation

```
> rollsd <- rollapply(datazoo, width =10, sd, by=1)
```

The same with separate weekly windows

```
> require(xts)
> rollsd <-  apply.weekly(datazoo, sd) ##daily, monthly, quarterly, yearly
```

Conversion to weekly data

```
> weeklydata <- apply.weekly(dailydata, last)  ## first, mean
```

apply functions are usually faster than traditional loops (for/while)

Rate of return / growth rate

Simple rate of return:

$$R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} = \exp(r_t) - 1$$

Logarithmic rate of return (=continuously compound interest rate:):

$$r_t = \ln(P_t + D_t) - \ln(P_{t-1}) = \ln(1 + R_t)$$

Rate of return / growth rate

Simple returns:

- Easy to calculate for a portfolio of assets
- Easy to communicate to non-statisticians
- Not symmetric nor additive...

Log returns

- Symmetric and additive
- Suitable for econometric modeling financial markets dynamics

Rate of return / growth rate

Simple return: $R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} = \exp(r_t) - 1$

Log-return: $r_t = \ln(P_t + D_t) - \ln(P_{t-1}) = \ln(1 + R_t)$

For portfolio of *K* assets:

$$R_{t,portfolio} = \sum_{k=1}^{K} w_k R_{t,k} = w' \boldsymbol{R_t}$$

$$r_{t,portfolio} \neq \sum_{k=1}^{K} w_k r_{t,k}$$

Descriptive statistics

Mean: $\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} r_t$

Variance: $\hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}(r_t - \hat{\mu})^2$

Standard deviation: $\hat{\sigma} = \sqrt{\hat{\sigma}^2}$

Skewness: $\hat{S} = \frac{\frac{1}{T}\sum_{t=1}^{T}(r_t - \hat{\mu})^3}{\hat{\sigma}^3}$

Kurtosis: $\hat{K} = \frac{\frac{1}{T}\sum_{t=1}^{T}(r_t - \hat{\mu})^4}{\hat{\sigma}^4}$

Theoretical moments for $r \sim N(0,1)$

Expected value: $\mu = E(r_t) = 0$

Variance: $\sigma^2 = E((r_t - \mu)^2) = 1$

Standard deviation: $\sigma = 1$

Skewness: $S = E((r_t - \mu)^3) = 0$

Kurtosis: $K = E((r_t - \mu)^4) = 3$

Financial series characteristics

1. Fat tails
   - Kurtosis above 3

2. Asymmetry of ups and downs (deeper declines)
   - Negative skewness

Data for WIG returns

(daily data from 10.02.2016 – 11.02.2021)

$\hat{\mu} = 0.000217$ → annualized return 0.054

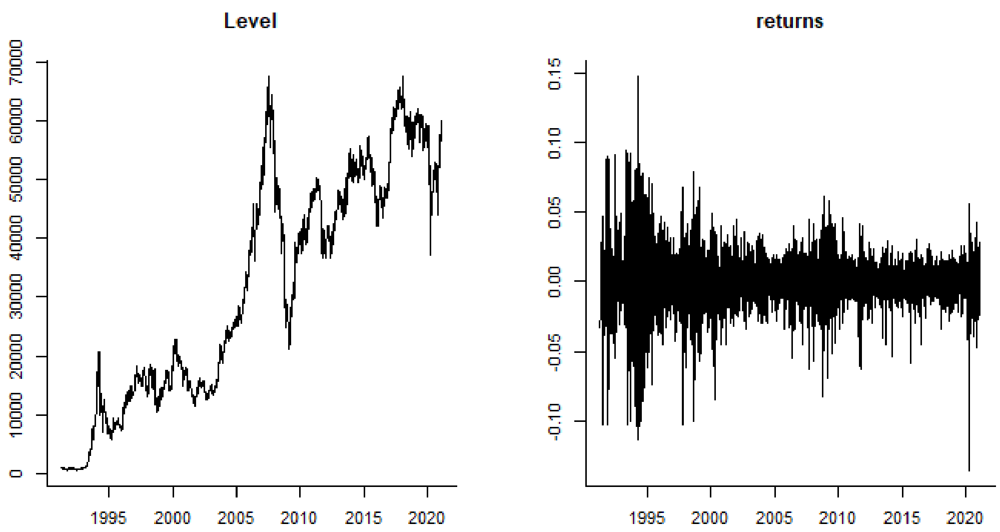
$\hat{\sigma} = 0.0115$ → annualized std. dev. 0.183

$\hat{S} = -1.45$

$\widehat{K} = 21.33$

Norte: Standard deviation vs synthetic risk index in SRRI in KIID (link)

Financial series characteristics



Grube ogony - testowanie

Testy

- D'Agostino: H0: $S = 0$
- Anscombe-Glynn: H0: $K = 3$
- Jarque-Bera: H0: $S = 0 \wedge K = 3$

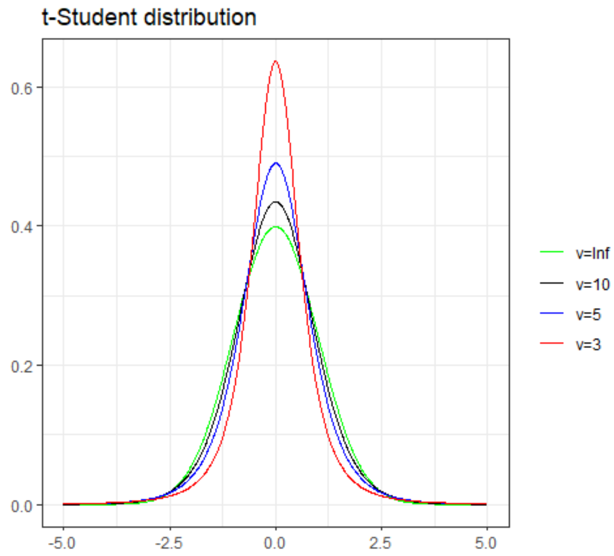
Fat tail: t-Student distribution

t-Student distribution:

- For $v = \infty$ normal distr.
- For $v < 2$ no variance
  [variance$\neq$ 1!!!]
- For assets usually $v \sim 5$

Variance: $Var(t_v) = \frac{v}{v-2}$

Kurtosis: $K(t_v) = 3 + \frac{6}{v-4}$

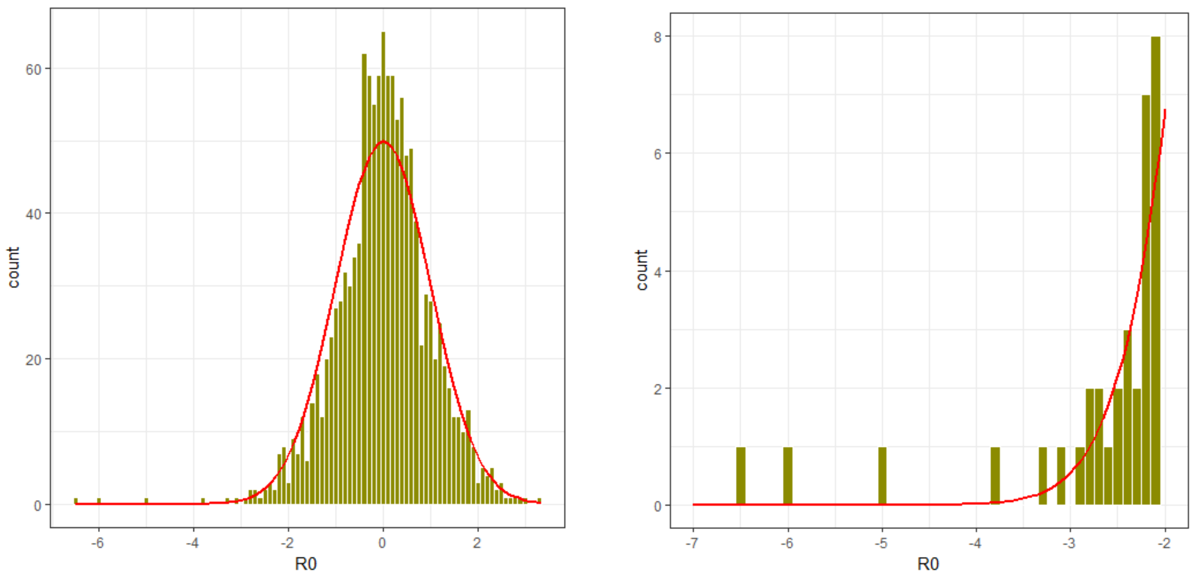


Fat tails: how to test

Figures

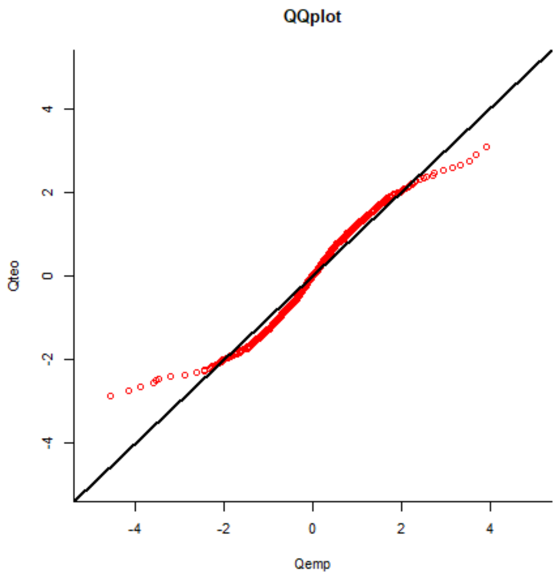
- QQ (quantile-quantile plot)
- Density plot

Empirical density vs normal distribution



QQ – plot (quantile-quantile plot)
theoretical quantile: normal distr.

	Qemp	Qteo
1%	-3.009	-2.326
2%	-2.387	-2.054
3%	-2.018	-1.881
4%	-1.750	-1.751
5%	-1.540	-1.645
6%	-1.426	-1.555
7%	-1.331	-1.476
8%	-1.259	-1.405
9%	-1.164	-1.341
10%	-1.093	-1.282
11%	-1.028	-1.227
12%	-0.982	-1.175
13%	-0.924	-1.126
14%	-0.875	-1.080
15%	-0.832	-1.036



Fat tail: t-Student distribution

Variance of t_v:

$$Var(t_v) = \frac{v}{v-2}$$

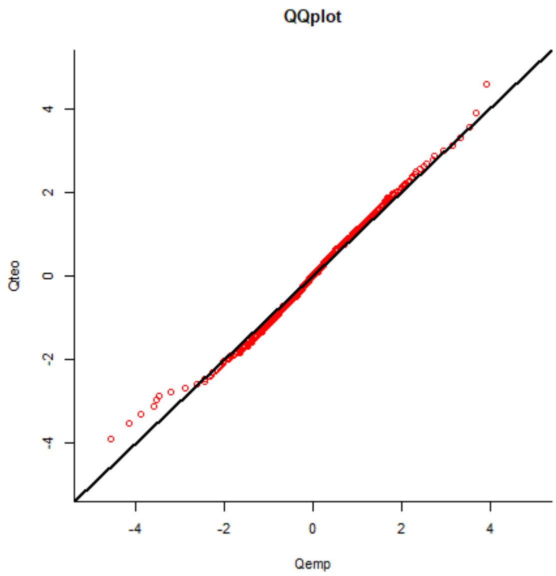
Quantile p for a variable with expected value μ and std. deviation σ

$$Q_p = \mu + \sigma\left(T_v^{-1}(p)\sqrt{\frac{v-2}{v}}\right)$$

Important!!!

Differences between R functions:

```
rt/qt/dt/ct                  – stats package
rdist/qdist/ddist/cdist – rugarch package
```

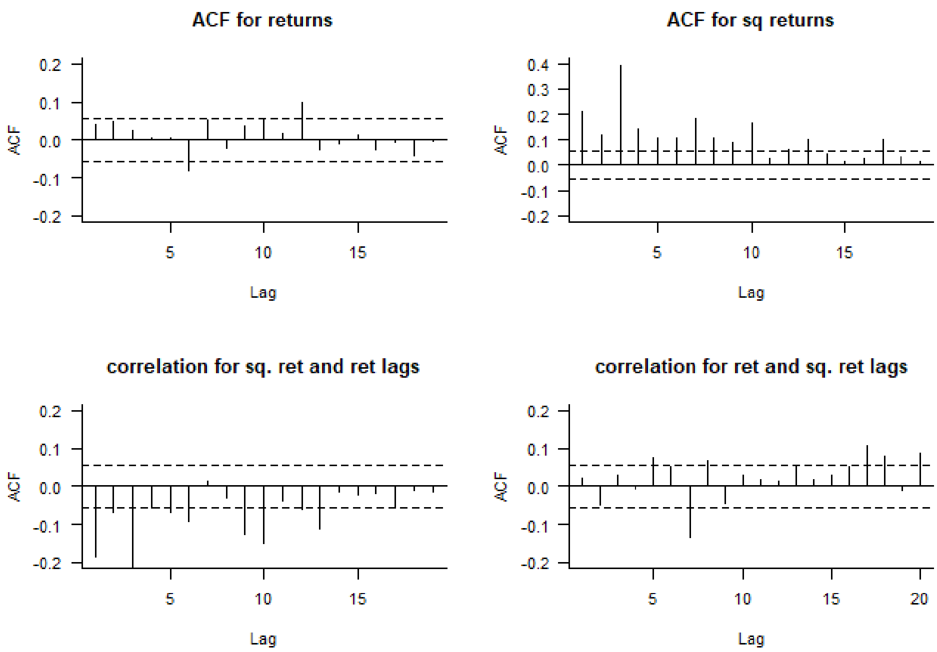


* More details in *Student t Distributed Linear Value-at-Risk* – link

Financial series characteristics

1. Fat tails
2. Asymmetry of ups and downs (deeper declines)
3. No autocorrelation of returns
   - $cor(r_t, r_{t-p}) = 0$
4. Non-linear autocorrelation dependencies
   - no autocorrelation $\neq$ independence
   - $cor(r_t^2, r_{t-p}^2) \neq 0$: volatility clustering
   - $cor(r_t^2, r_{t-p}) \neq 0$: leverage effect
   - ale $cor(r_t, r_{t-p}^2) = 0$

Financial series characteristics: WIG



Ljunga-Boxa test (adjusted portmanteau)

Test for autocorrelation of order H with null:

$$H0: \rho_1 = \rho_2 = \cdots = \rho_H = 0$$

Test statistics:

$$Q_{LB} = T(T+2)\sum_{h=1}^{H} \frac{\hat{\rho}_h^2}{T-h}$$

Under the null H0 statistics Q_{LB} is $\chi^2(H)$ distributed

Wyniki dla WIG:

```
data: y0;   LB = 45.034, df = 20, p-value = 0.001092
data: y0^2; LB = 1385.4, df = 20, p-value < 2.2e-16
```

Topic 2: Exercises

Exercise 2.1. Draw QQplot vs normal distribution for the below data:

0.49	-0.56	0.61	0.67	0.82	0.85	-2.04	-0.65	0.80	-1.00

knowing that the quantiles of normal distribution are:

q	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
qnorm(q)	-1.64	-1.04	-0.67	-0.39	-0.13	0.13	0.39	0.67	1.04	1.64

Exercise 2.2. For selected investment fund returns:

a. Calculate: mean, std. dev., skewness and kurtosis (annualized)

b. Verify if skewness is null and kurtosis equal to 3

c. Standardize returns (R^*)

d. Compare empirical density of R^* to the pdf of normal distribution

e. Draw QQ plot vs normal distribution

f. Estimate t-Student parameters (degree of freedom)

g. Draw QQ plot vs t-Student pdf

h. Plot ACF to visualize if:

$$cor(r_t, r_{t-p}) = 0;\ cor(r_t^2, r_{t-p}^2) \neq 0;\ cor(r_t^2, r_{t-p}) \neq 0 \text{ and } cor(r_t, r_{t-p}^2) = 0$$

i. Check for autocorrelation of returns and their squares with the LB test

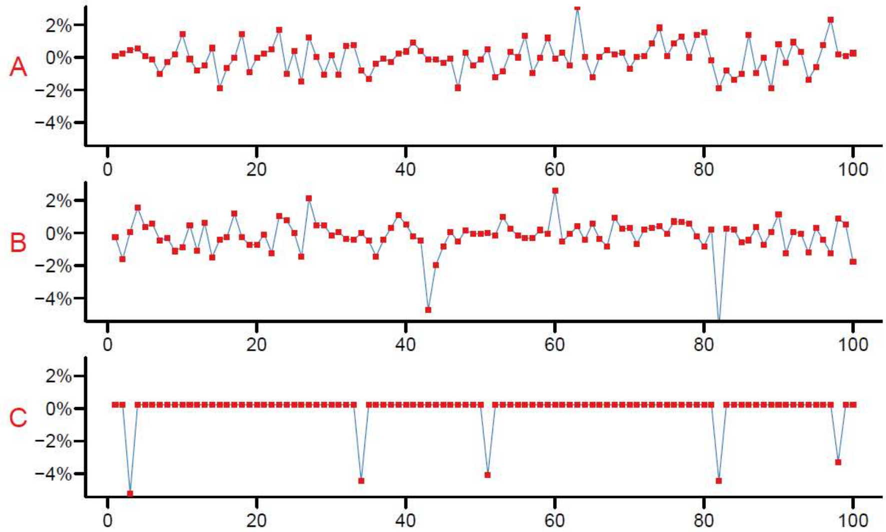
Topic 3

Risk measures: VaR and ES

- Value at Risk(VaR) and Expected Shortfall (ES) definitions
- Stages of VaR and ES calculation
- Metods of estimating VaR and ES
- Historical simulation
- Parametric models for VaR and ES
- Monte-Carlo simulation
- Cornish-Fisher expansion

Risk ≠ standard deviation

Three series with $E(Y) = 0$ i $Sd(Y) = 1$ (Danielson, 2011)



Risk: Value at Risk and Expected Shortfall

Value at Risk, VaR:

Definition 1: $P(r \leq VaR_p) = p$

Definition 2: $p = \int_{-\infty}^{VaR_p} f(r)dr$

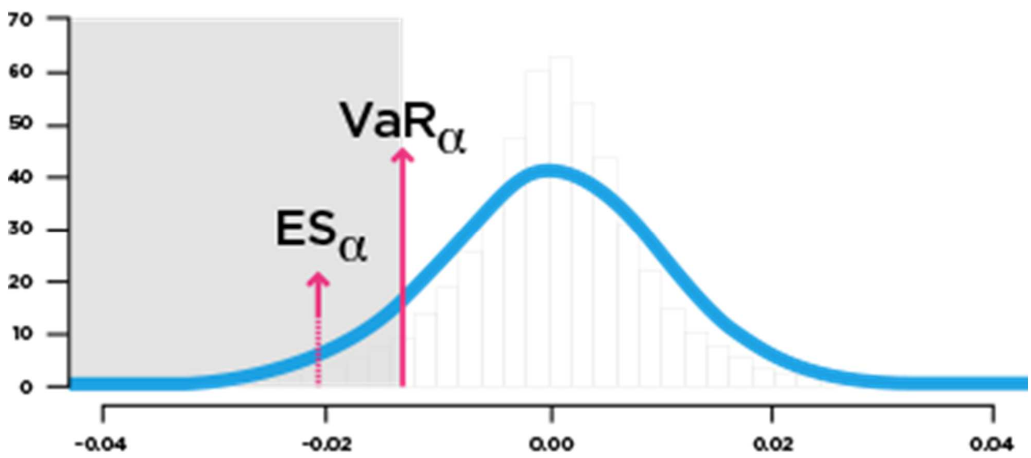
Expected shortfall, ES:

Definition 1: $ES_p = E(r|r \leq VaR\_p)$

Definition 2: $ES_p = \frac{1}{p}\int_{-\infty}^{VaR_p} rf(r)dr$

Definition 3: $ES_p = \frac{1}{p}\int_0^p VaR_s\, ds$

Value at Risk and Expected Shortfall



Value at Risk and Expected Shortfall

VaR/ES calculation stages

1. Setting tolerance level: p
2. Setting horizon: H
3. Choosing estimation sample period $1{:}T$
4. Choosing a model + backtesting method
5. VaR/ES computation (for period $T+1$)

Basel ii/iii: VaR as a Risk measure (link, p. 44)
Basel iv: plans to chanes into ES (link, p. 52)

Value at Risk: Basel II

Quantitative standards Basel II

a. 99th percentile V*aR* must be computed on a daily basis
b. In calculating VaR the minimum *"holding period"* will be 10 trading days.
   Banks may use VaR numbers calculated according to shorter holding periods scaled up to ten days by the square root of time
c. The choice of sample period for calculating VaR is constrained to a minimum length of one year.
d. banks will be free to use models based, for example, on variance-covariance matrices, historical simulations, or Monte Carlo simulations
e. The multiplication factor will be set by individual supervisory authorities on the basis of their assessment of the quality of the bank's risk management system, subject to an absolute minimum of 3. Banks will be required to add to this factor a "plus" directly related to the ex-post performance of the model, thereby introducing a builtin positive incentive to maintain the predictive quality of the model. The plus will range from 0 to 1 based on the outcome of so-called "backtesting."

Source: Basle Committee on Banking Supervision, 1996.
AMENDMENT TO THE CAPITAL ACCORD TO INCORPORATE MARKET RISKS (link, s. 44)

VaR and ES calculation methods

A. Parametric / non-parametric models

B. Analytical formula / Monte-Carlo simulations

C. Conditional / unconditional volatility

A. Non-parametric model: historical simulation

- We assume that the distribution of returns is well approximated by past/historical returns
- We sort past returns from the lowest to highest:

$$rs_1 < rs_2 < \cdots < rs_N$$

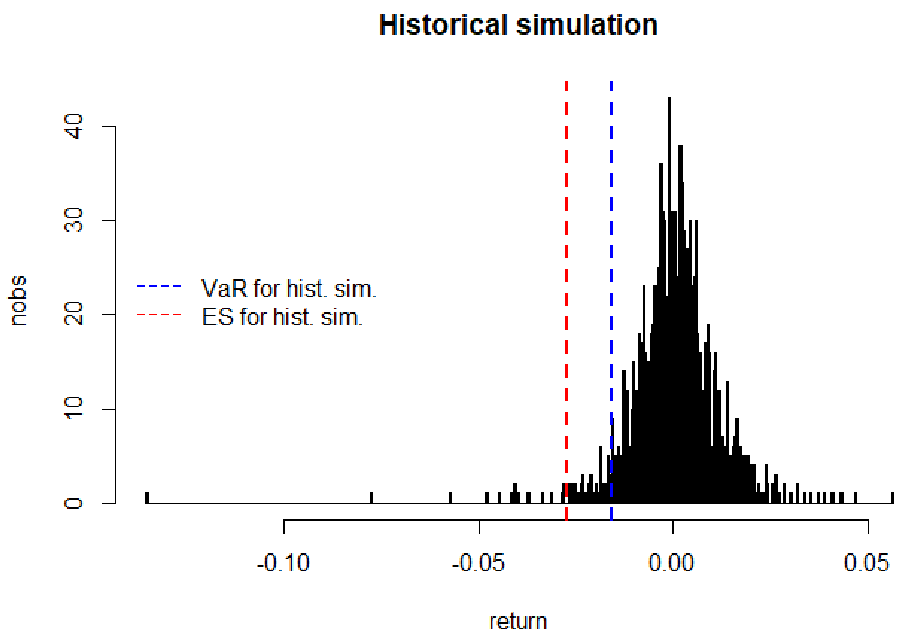
- VaR_p is equal to p-th quantile of distribtion, so that for $M = floor(pN)$

$$VaR_p = rs_M$$

- ES_p is equal to the average of the worst returns lower than VaR_p

$$ES_p = \frac{1}{M}\sum_{1}^{M} rs_i$$

A. Non-parametric model: historical simulation for WIG



A. Non-parametric model: historical simulation

HS shortcomming: low precision of VaR, especially for low p!

Reason: we only use information about one quantile and not entire distribution

Std. dev. for p-th quantile is : $S(VaR_p) = \sqrt{\frac{p(1-p)}{Tf^2}}$

For WIG:

$$p = 0.05; \quad N = 2587; \quad f = 4.98; \quad S(VaR_p) = 0.00086$$

95% confidence interval:

$$P\{VaR_p \in (\widehat{VaR}_p - 1.96S(VaR_p); \widehat{VaR}_p + 1.96S(VaR_p))\} = 0.95$$

For WIG:

$$P(VaR_p \in (-0.0213; -0.0179)) = 0.95$$

A. Non-parametric model: historical simulation

Pros:

- Simplicity
- Easy to communicate
- No need to make assumptions
- Extension possibilities (e.g. for volatility clustering)

Cons:

- Full dependence on historical data
- Difficult to conduct counterfactual caluclations
- Low precision of VaR estimates

B. Parametric models

- We search for the distribution (pdf) of future returns: $f(r)$
- Knowing this distribution allows to calculate VaR and ES

$$p = \int_{-\infty}^{VaR_p} f(r)dr$$

$$ES_p = \frac{1}{p}\int_{-\infty}^{VaR_p} rf(r)dr$$

B. Parametric models: normal distribution

If $r \sim N(\mu, \sigma^2)$ then:

$$VaR_p = \mu + \sigma\Phi^{-1}(p)$$
$$ES_p = \mu + \sigma\frac{\phi(\Phi^{-1}(p))}{p}$$

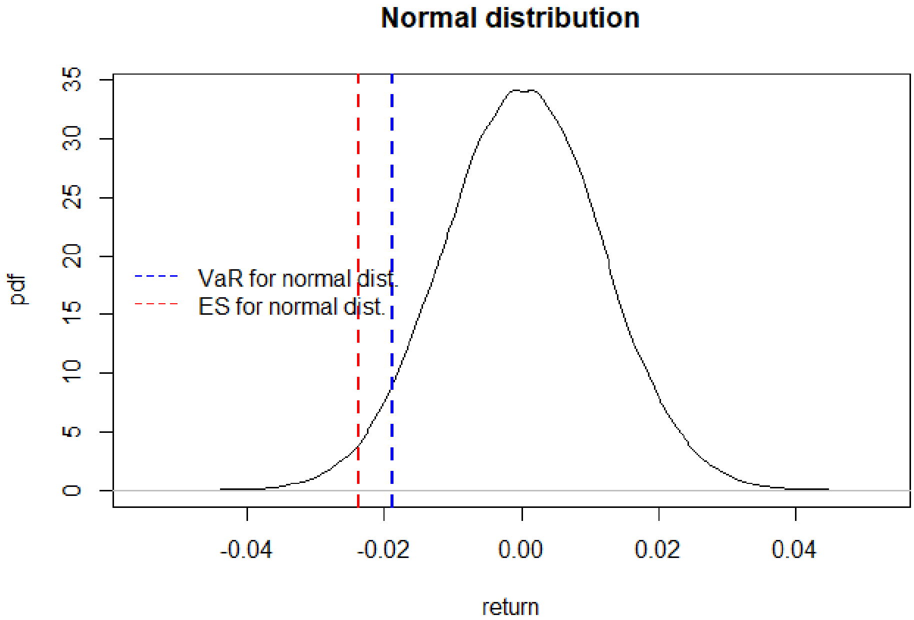
where Φ i ϕ are pdf and cdf for $N(0,1)$

Tables for $\mathrm{r} \sim N(0,1)$ are (with minus)

p	0.5	0.1	0.05	0.025	0.01	0.001
VaR	0	1.282	1.645	1.960	2.326	3.090
ES	0.798	1.755	2.063	2.338	2.665	3.367

Note: we assume $r_{T+1} \sim N(\mu, \sigma^2)$ and calculate VaR_{T+1} and ES_{T+1}

B. Parametric models: normal distribution



C. Monte Carlo simulations

- Assume we know DGP but cannot derive analitical formula for VaR/ES
- We can resort to Monte Carlo simulations.
- MC steps:
  1. Create „N" artificial observations from known DGP:
     $r^{(n)}$ for $n = 1,2, \ldots, N$
  2. Sort artifical returns from lowers to highest:
     $rs^{(1)} \leq rs^{(2)} \leq \ \ldots$
  3. Set $M = \text{floor}(pN)$ and calculate:

$$VaR_p = rs^{(M)} \text{ and } ES_p = \frac{1}{M}\sum_1^M rs^{(i)}$$

C. Monte Carlo simulations

MC vs analytical calculations comparison for WIG:

- metoda parametryczna vs. MC dla rozkładu normalnego ($N = 100\ 000$)

VaR

Analytical method: -0.02091533

MC simulations: -0.02095668

ES:

Analytical method : -0.02624805

MC simulations : -0.02626429

Fat tails

Two methods to account for „fat tails”:

- *t*-Student distribution
- Cornisha-Fisher expansion:
  correction of quantiles from normal distribution for skewness and kurtosis

More sophisticated methods (beyond this course):

- EVT, extreme value theory

Fat tails: t-Student distribution

Reminder:

Variance t_v: $Var(t_v) = \frac{v}{v-2}$

Quantile p: $Q_p = \mu + \sigma\left(T_v^{-1}(p)\sqrt{\frac{v-2}{v}}\right)$

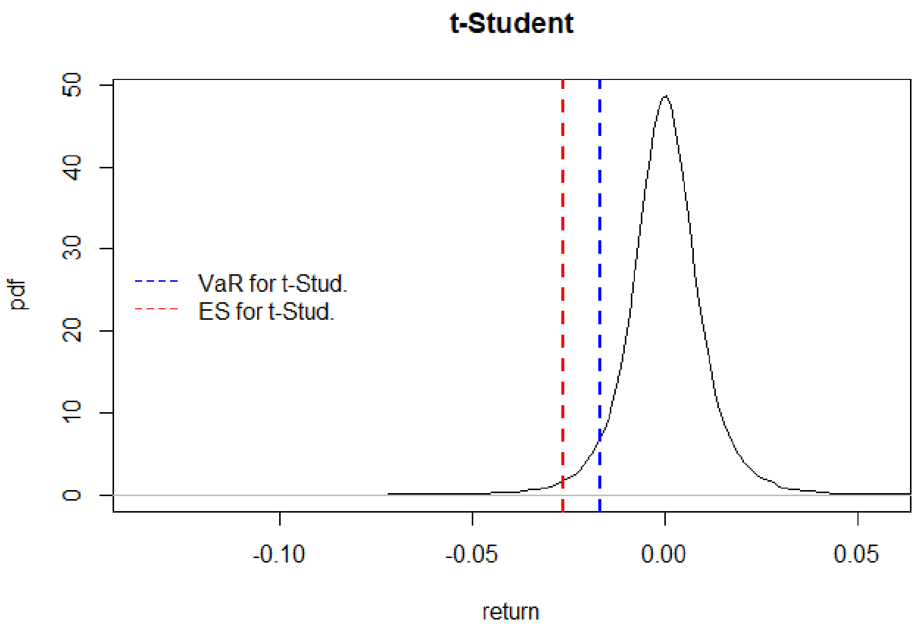
VaR:

$$VaR_p = \mu + \sigma\left(T_v^{-1}(p)\sqrt{\frac{v-2}{v}}\right)$$

ES, numerical integration:

$$ES_p = \mu + \sigma\frac{1}{p}\int_0^p\left(T_v^{-1}(s)\sqrt{\frac{v-2}{v}}\right)ds = \frac{1}{p}\int_0^p VaR_s\, ds$$

Fat tails: t-Student distribution



Fat tails: Cornish-Fisher expansion

- Cornish-Fisher expansion accounts for skewness and kurtosis (also higher moments*) in quantile calculations:

$$VaR_p = \mu + \sigma\left(\gamma_p + \frac{\gamma_p^2 - 1}{6}S + \frac{\gamma_p^3 - 3\gamma_p}{24}(K-3) - \frac{2\gamma_p^3 - 5\gamma_p}{36}S^2\right)$$

  where $\gamma_p = \Phi^{-1}(p)$.

- For normal distribution ($S = 0$ and $K = 3$), the formula simplifies to:

$$VaR_p = \mu + \sigma\gamma_p$$

* More on Cornish-Fisher expansion – link

** MRM methodology - link

Fat tails: Cornish-Fisher expansion WIG example

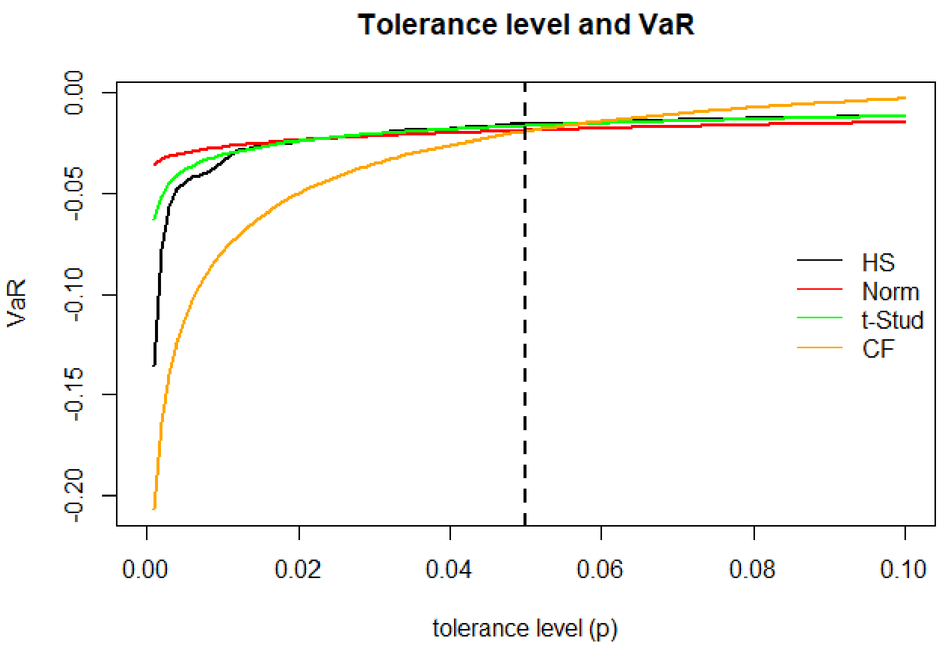
$$S = -0.450$$

$$K = 7.055$$

$$\gamma_p = -1.645$$

$$\left(\gamma_p + \frac{\gamma_p^2 - 1}{6}S + \frac{\gamma_p^3 - 3\gamma_p}{24}(K-3) - \frac{2\gamma_p^3 - 5\gamma_p}{36}S^2\right) = -1.687$$

Models comparison



Topic 3: Exercises

Exercise 3.1. The distribution of log-returns are t-Student with 5 degrees of freedom. Compute VaR for a selected tolerance level p if expected return is 0.5% and std. dev. amounts to 6%.
Critical values for t-Student with v =5 are equal to:

1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
-3.365	-2.757	-2.422	-2.191	-2.015	-1.873	-1.753	-1.649	-1.558	-1.476

Note: critical values were generated using function `qt(p,5)`

Exercise 3.2. We know that returns are uniformly distributed over the interval (-0.01;0.01), $r \sim U(-0.01{,}0.01)$. Calculate VaR and ES for p=0.05 and p=0.10.

Exercise 3.3. Calculate VaR using Cornish-Fisher expansion if returns moments are as follows: $\mu = 0.5\%$, $\sigma = 5\%$, $S = -1$, $K = 7$. Assume the tolerance level at $p = 0.05$ or 0.025.
[$\Phi^{-1}(0.05) = -1.645$ and $\Phi^{-1}(0.025) = -1.960$]

Exercise 3.4*. Create a function in R, which will allow you to compute ES consistent with the Cornish-Fisher expansion. Use the function to compute ES for $p = 0.05$ or 0.025 and moments from exercise 3.3.

Temat 3: Exercises

Exercise 3.5.
For return of the selected investment fund (from Exercise 2.2), do the following::

a. Consider which of the four methods discussed so far (HS, normal, t-Student, CF) you think is appropriate for VaR calculation
b. Calculate the VaR and ES values on the basis of the above 4 methods for a tolerance level of 5%. Why do the results differ?
c. Create a plot for the empirical density function, the density of the normal and t-Student distribution. Plot the values from point b on the graph.
d. Calculate the VaR and ES values on the basis of the above 4 methods for a tolerance level of 1% and compare tchem with values from point b.
e. Discuss the obtained results

Topic 4

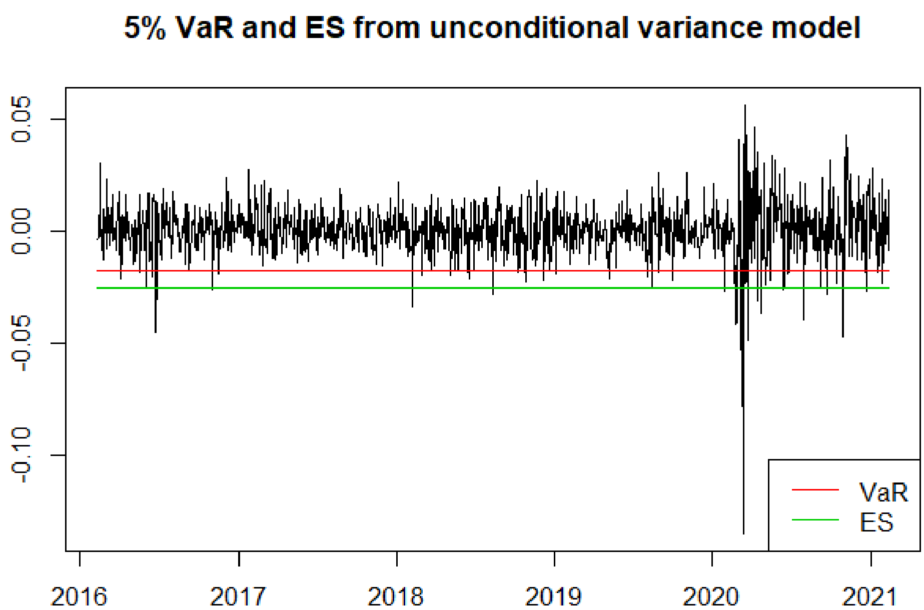
Volatility clustering

- Volatility clustering
- Moving average (MA)
- Exponentially Weighted Moving Average (EWMA)
- GARCH model

Financial series characteristics

1. Fat tails
2. Asymmetry of ups and downs (deeper declines)
3. No autocorrelation of returns
   - $cor(r_t, r_{t-p}) = 0$
4. Non-linear autocorrelation dependencies
   - $cor(r_t^2, r_{t-p}^2) \neq 0$: volatility clustering
   - $cor(r_t^2, r_{t-p}) \neq 0$: leverage effect

Volatility clustering and VaR/ES from t-Student distribution



Methods of volatility modelling

- MA: Moving Average
- EWMA: Exponentially Weighted Moving Average
- GARCH model: *Generalized Autoregressive Conditional Heteroskedasticity*
- SV: Stochastic Volatility
- IV: Implied Volatility

Important:

$$VaR_{T+1|T} = VaR(\mu_{T+1|T}, \sigma_{T+1|T}, \dots)$$

hence we need to calculate a forecast $\sigma_{T+1|T}$

A. Moving average

Formula for variance forecast (T – moment forecast formulation):

$$\sigma^2_{T+1} = \frac{1}{W} \sum_{s=0}^{W-1} (r_{T-s} - \mu)^2$$

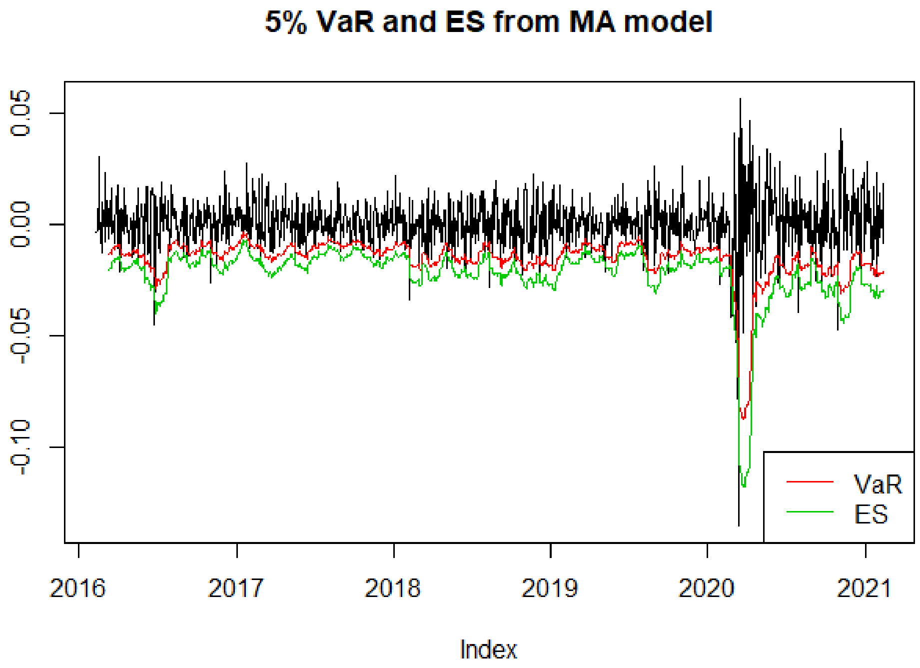
Note 1: the value depends on window length W

Note 2: we use information up to moment T

Note 3: the above formula can be written down as weighted average with equal weights

$$\sigma^2_{T+1} = \sum_{s=0}^{W-1} w_s (r_{T-s} - \mu)^2, \text{where } w_s = \frac{1}{W}$$

A. VaR and ES from moving average model



B. Exponentially Weighted Moving Average, EWMA

- Variance forecast calculated as a weighted average of past observations:

$$\sigma_{T+1}^2 = \sum_{s=0}^{\infty} w_s (r_{T-s} - \mu)^2$$

in which weights form a geometric sequance:

$$w_s = \lambda w_{s-1} \qquad \leftrightarrow \qquad w_s = \lambda^s \times \frac{1-\lambda}{\lambda}$$

Note: given that weights sum to unity, this implies that $w_0 = 1 - \lambda$

$$\sigma_{T+1}^2 = (1-\lambda)(r_T - \mu)^2 + \lambda \sigma_{t-1}^2$$

- In RiskMetrics (JP Morgan, link) parameters λ and μ are not estimated but calibrated. For daily data the proposed values are:

$$\lambda = 0.94 \text{ and } \mu = 0$$

VaR and ES from EWMA

- EWMA model:

$$r_t \sim D(0, \sigma_t^2)$$
$$\sigma_t^2 = (1-\lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2$$

- Variance forecast:

$$\sigma_{T+1}^2 = (1-\lambda) r_T^2 + \lambda \sigma_T^2$$

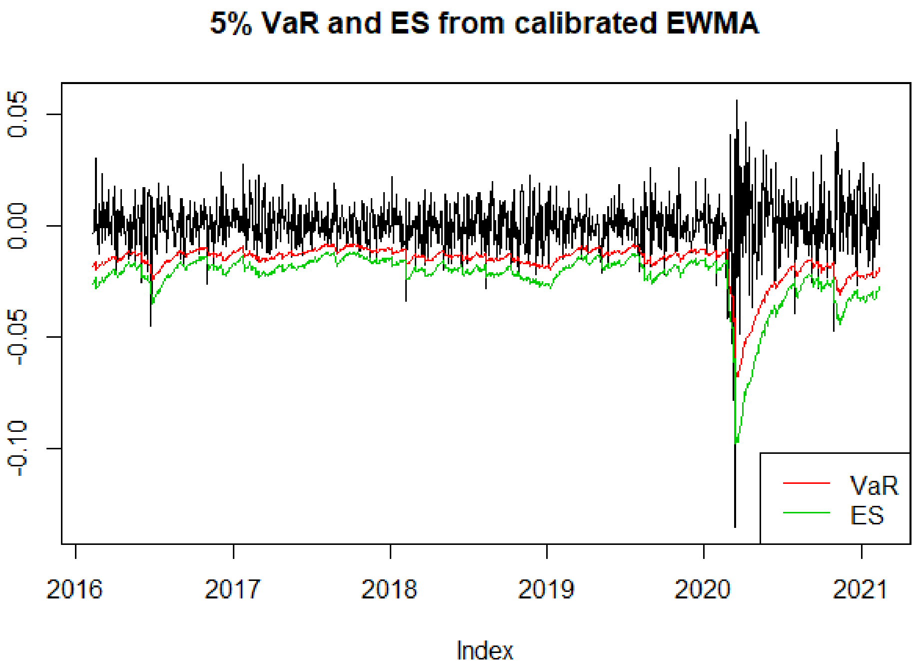
- Let F_D be the cdf of D distribution, which implies that:

$$VaR_{p,T+1} = \sigma_{T+1} F_D^{-1}(p)$$

- ES amounts to:

$$ES_{p,T+1} = \sigma_{T+1} \frac{1}{p} \int_0^p F_D^{-1}(s) ds$$

VaR and ES from EWMA



GARCH as EWMA extension

- RiskMetrics:

$$r_t \sim D(0, \sigma_t^2)$$
$$\sigma_t^2 = (1-\lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2$$

- GARCH(1,1):

$$r_t = \mu + \epsilon_t, \qquad \epsilon_t \sim D(0, \sigma_t^2)$$
$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2$$

- EWMA restrictions (EWMA=Integrated GARCH, IGARCH):

$$\mu = 0; \omega = 0; \alpha = 1 - \lambda; \beta = \lambda$$

C. GARCH models

- Benchmark GARCH(1,1) model specification:

$$r_t = \mu + \epsilon_t, \qquad \epsilon_t \sim D(0, \sigma_t^2)$$
$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

  where $\omega > 0$ and $\alpha, \beta \geq 0$.

- Equilibrium (unconditional) variance is:

$$\bar{\sigma}^2 = \frac{\omega}{1 - (\alpha + \beta)}$$

- Extensions:
  - Leverage effect: EGARCH, GJR-GRACH
  - Risk premium: GARCH-in-Mean

VaR and ES from GARCH

- GARCH model:

$$r_t = \mu + \epsilon_t, \qquad \epsilon_t \sim D(0, \sigma_t^2)$$
$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

- Variance forecast:

$$\sigma_{T+1}^2 = \omega + \alpha\epsilon_T^2 + \lambda\sigma_T^2$$

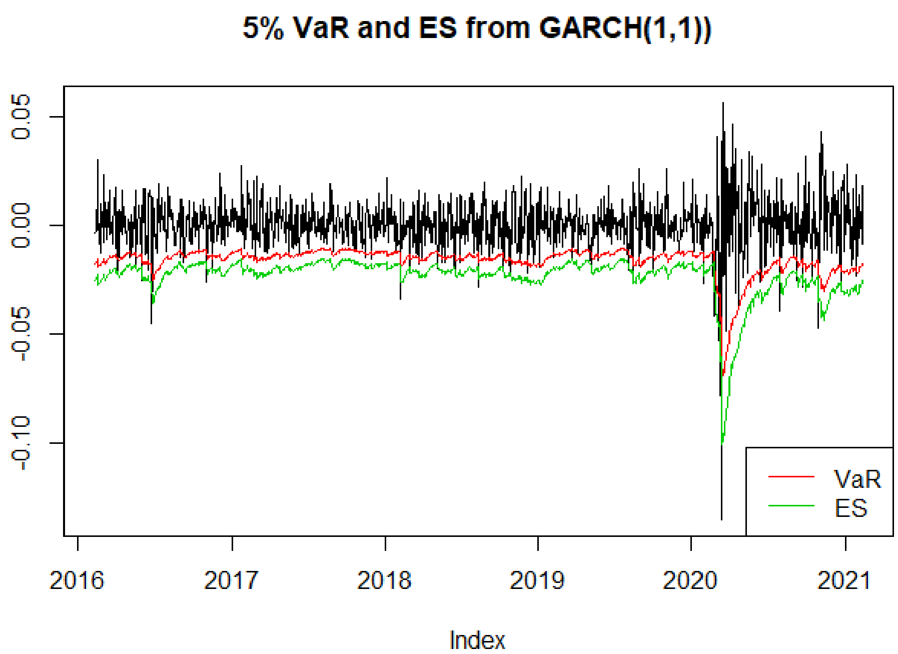
- Let F_D be the cdf of D distribution, which implies that:

$$VaR_{p,T+1} = \mu + \sigma_{T+1} F_D^{-1}(p)$$

- ES amounts to:

$$ES_{p,T+1} = \mu + \sigma_{T+1} \frac{1}{p} \int_0^p F_D^{-1}(s) ds$$

C. VaR and ES from GARCH model



Veryfying GARCH models

- To veryfy the quality of GARCH model we test standarized residuals:

$$u_t = \frac{\epsilon_t}{\sigma_t}$$

- The should be characterized by:
  - no autocorrelation
  - no autocorrelation of squares
  - QQ plot should indicate that the assumed distribution is correct

$$u_t \sim IID\ D(0,1)$$

Topic 4: Exercises

Exercise 4.1. The model for the rate of return (expressed as %) is:

$$r_t = 0.1 + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma_t^2)$$
$$\sigma_t^2 = 0.4 + 0.1\epsilon_{t-1}^2 + 0.8\sigma_{t-1}^2$$

It is known that $r_T = -0.9$ and $\sigma_T^2 = 4$.

1. Calculate a forecast for the moments μ_{T+1} and σ_{T+1}
2. Copute VaR_{T+1} and ES_{T+1} for toletance level $p = 5\%$
3. What is the equilibrium variance in this model?

Exercise 4.2. For your chosen asset:

1. Compute variance forecast σ_{T+1}^2 using discussed methods (constant variance, MA, EWMA, GARCH).
2. Assume t-Student distribution with $v = 5$ and compute VaR/ES for the above methods (for tolerance level $p = 1\%$ and $p = 5\%$)
3. Repeat points 1 and 2 for notmal distribution
4. Create a table with the results

Topics 1-4 presentation

Content of the presentation:

a. <1.0p> Information about the fund (KIID), including fees

b. <1.5p> Historical data + returns characteristics
(moments, QQ plot, density plot)

c. <1.5p> GARCH model estimates (+ selected plots)

d. <3.0p> VaR and ES (1% i 5%) calculated with:

- Historical simulation
- Parametric method (normal / t-Student)
- Cornish-Fisher expansion
- EWMA
- GARCH

Note: all results should be presented in one table.

e. <1.0p> A plot: VaR vs tolerance level for 5 above methods

f. <1.0p> General discussion about the risk of investing in a given fund

Additionally, 1 p. for the quality of presentation and the speech. Time limit: 5 minutw. Avoid a large number of slides (7 slides is a good choice). Presentation in pdf file entitled *SurnameName.pdf* download to MT.

Topic 5

VaR and ES for longer horizons

- *Ssquare root of time* method
- Cornish-Fisher expansion for $H>1$
- Monte Carlo simulations
- Bootstrap
- H-period returns

Topic 6

Stress tests

- Stress test and VaR/ES
- Sensitivity analysis
- Scenario analysis
- Historical and hypothetical scenarios
- Stressed-VaR

Topic 7

Backtesting

- Backtesting procedure
- VaR violations and tolerance level
- Binomial distribution
- Traffic lights method
- Kupiec test
- Christoffersen tests
- Tests power
- McNeil and Frey test for ES

Topics 5-7 presentation

Contents of the presentation:

a. <0p> Remain main informations about the fund.

b. <1.5p> Present 5% VaR for horizons from 1 to 4 days using:

- square root of time method (normal distribution)
- Cornish-Fisher expansion
- MC simulations from GARCH model

Present the results in a Table and on the graph..

c. <2p> Compute the risk in 1-year horizon by comparing VaR and S-VaR (for stressed values of variance computed as 99 percentile from 21-day window) for 5% tolerance level and assuming normal distribution. Present the histogram of variance with the stressed and unconditional variance values.

d. <2.5p> Compute % change in your fund portfolio value in the scenario of the initial months of COVID-19 pandemics:

- commodity price decline by 50%, but precious metals price increase by 25%
- stock price declines by 10% in developed countries and 15% in emerging economies (Note: PL is classified as EME)
- depreciation of EME currencies by 10%
- yield curve downward shift by 100 bp.

Present the structure of your portfolio, sensitivity analysis and the calculated impact on portfolio value. Compare the results with realized change in the fund value in the period: 1.03-31.05.2020.

e. <3p> Backtest 1% VaR with evaluation window of 250 observations (traffic lights, Kupiec test, Christoffersen, McNeil-Frey) for:

- Historical simulation / normal distribution
- Cornish-Fisher
- EWMA or GARCH

Present p-values of tests as well as VaR exceedance plots. Which model is the best?

Additionally, 1 p. for the quality of presentation and the speech. Time limit: 5 minutw. Avoid a large number of slides (7 slides is a good choice). Presentation in pdf file entitled *SurnameName.pdf* download to MT.