MATERIALS FOR THE COURSE MODELLING FINANCIAL RISK WITH R

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About

This script contains materials for the course Modelling financial risk with R.

The course also contains R codes that can be found on the page: http://web.sgh.waw.pl/~mrubas/

As additional materials we recommend:

- Jon Danielsson 2011. "Financial Risk Forecasting", Wiley https://www.financialriskforecasting.com/
- Alexander C., 2009. "Market Risk Analysis", Wiley

Topic 1

Introduction

- Course requirements
- Additional material
- R package

Aims

Block 1

- 1. Discussing financial series characteristics
- 2. Presenting financial time series models
- 3. Prezentacja metod liczenia VaR

Block 2

- 1. Backtesting
- 2. Stress tests

Additionally

- 1. Programming in R
- 2. Developing presentation and public speech skills

Materials

Main materials:

- Script
- R codes

Available at course page: web.sgh.waw.pl/~mrubas

Recommended books: Danielsson J. 2011. Financial Risk Forecasting, Wiley

Dowd K., 2005. Measuring Market Risk, Wiley Alexander C., 2009. Market Risk Analysis, Wiley Jorion P., 2007. Value at risk. McGraw-Hill

Internet resources:

RiskMetrics – technical document: link



Meetings outline

Block 1

- i. Introduction to R
- ii. Time series in R (zoo, Quandl, apply, ggplot2)
- iii. Financial time series characteristics
- iv. VaR & ES: unconditional distribution models
- v. VaR & ES: volatility clustering (EWMA and GARCH)
- vi. Presentations

Block 2

- i. VaR & ES for longer horizons
- ii. Backtesting
- iii. Stress tests
- iv. Presentations

Grades

Points are attributed for:

- 20 points for 2 presentations
- 10 points for the exam
- 2 points for activity

points	≤ 15	≤ 18	≤ 21	≤ 24	≤ 27	>27
grade	2.0	3.0	3.5	4.0	4.5	5.0

What is R

- Environment for statistical calculations and visualization of results, created by Robert Gentleman and Ross Ihaka at the University of Auckland in 1996. The name comes from the first letters of the authors' names and is a reference to the S language
- GNU R is distributed as source code and in binary form with many distributions for Linux, Microsoft Windows and Mac OS
- R is used in many well-known companies, including Facebook, Google, Merck, Altera, Pfizer, LinkedIn, Shell, Novartis, Ford, Mozilla and Twitter.
- Producers of commercial statistical packages (SPSS, SAS, Statistica) offer dedicated mechanisms ensuring their cooperation with R
- R provides a wide range of statistical techniques (linear and nonlinear modeling, classical statistical tests, time series analysis, classification, clustering, ...) and graphical.
- In addition, R is extendable with additional packages and user-written scripts.
- * On the basis of information from Wikipedia

Why R

1. Popularity

R is also the name of a popular programming language used by a growing number of data analysts inside corporations and academia



Dlaczego R

2. Comprehensiveness

"The great beauty of R is that you can modify it to do all sorts of things," said Hal Varian, chief economist at Google. "And you have a lot of prepackaged stuff that's already available, so you're standing on the shoulders of giants."

3. Price

"R first appeared in 1996, when the statistics professors Ross Ihaka and Robert Gentleman of the University of Auckland in New Zealand released the code as a **free software package**."



R – links

Webpage of R project

https://www.r-project.org/

Materials:

P. Kuhnert & B. Venables, <u>An Introduction to R: Software for Statistical Modeling & Computing</u>
P. Biecek, <u>Przewodnik po pakiecie R</u>
Rproject, <u>An Introduction to R</u>

Fxercise 1.1. 1. Download and unzip to folder Rcodes/funds investment funds prices from bossa.pl http://bossa.pl/pub/fundinwest/mstock/mstfun.zip http://bossa.pl/pub/ofe/mstock/mstfun.lst 2. Select 2-3 funds of different characteristics with price history of at least 5 years 3. Analyze the profile of these funds using Key Investor Information Document - KIID (kluczowe informacje dla inwestorów)

Topic 2

Financial times series

- $\bullet\,$ Downloading financial series to R
- \bullet zoo package in R
- Simple and logarithmic rate of return
- Moments of returns distribution
- Financial series characteristics
- QQ plot
- t-Student distribution

Ir	nporting financial time series
Q(> : > (>]	uandl package require(Quandl) cpiUS <- Quandl("FRED/CPIAUCNS", type = "zoo") ## CPI USA brent <- Quandl("EIA/PET_RBRTE_M", type = "zoo") ##ceny ropy brent
Q(> : > :	uantmod: Yahoo, Google, Oanda, require(quantmod) getSymbols("SPY", src = "yahoo")
Im	nporting from local files: csv, xls, xlsx, xml,
In	teraction with popular databases: MySQL, PostgreSQL, MS SQL Server,
Ti	ime series in R, zoo
Ті	ime series in R, zoo Time Series – TS – is a series of values $X_{t_1}, X_{t_2}, X_{t_3},;$ where $t_1 < t_2 < t_3 < \cdots$ are ordered time indices.
ті •	ime series in R, zoo Time Series – TS – is a series of values $X_{t_1}, X_{t_2}, X_{t_3},;$ where $t_1 < t_2 < t_3 < \cdots$ are ordered time indices. R packages to work with TS: tseries, timeSeries, tis, stats, zoo , xts,
ті •	<pre>ime series = TS = is a series of values X_{t1}, X_{t2}, X_{t3},; where t1 < t2 < t3 < are ordered time indices.</pre> R packages to work with TS: tseries, timeSeries, tis, stats, zoo, xts, zoo objects consist of coredata (vector or matrix) and time index:

Dates

Date object represents daily data as the numer of days from 01-01-1970

- > mydate <- as.Date("01-01-1970", format = "%d-%m-%Y")</pre>
- > weekdays(mydate) ##months(mydate) quarters(mydate)
- > mydate + 1
- > mydate <- mydate 5

difftime objects

```
> mydate1 <- as.Date("01-11-1990", format = "%d-%m-%Y")
> mydate - mydate1
```

Sequence of dates

- > seq(from=mydate, to=mydate1, by="5 months")
- > seq(from=mydate, by="2 months", length.out=20)

lubridate package helps to work with dates

```
> dmy("01-01-1970") + years(2)
```

```
> dmy("01-01-1970") + (0:19)*months(2)
```

> wday(mydate)

zoo objects

Merging objects

```
> merge(ts.zoo.1, ts.zoo.2) ## full merge
```

> merge(ts.zoo.1, ts.zoo.2, all=FALSE) ## inner merge

Windows

```
> window(ts.zoo, start=as.Date("2007-01-05"), end=as.Date("2008-02-01"))
```

Lags and leads

```
> lag(ts.zoo, -1) ## previous value
```

> lag(ts.zoo, 1) ## next value

Differences

>diff(ts.zoo)

Rates of returns

```
> diff(ts.zoo)/lag(ts.zoo, -1) ## simle
> diff(log(ts.zoo)) ## log-returns
```



Descriptive statistic	CS	
Mean:	$\hat{\mu} = \frac{1}{T} \sum_{t=1}^{T} r_t$	
Variance:	$\hat{\sigma}^2 = \frac{1}{T} \sum_{t=1}^T (r_t - \hat{\mu})^2$	
Standard deviation:	$\hat{\sigma} = \sqrt{\hat{\sigma}^2}$	
Skewness:	$\widehat{S} = \frac{\frac{1}{T} \sum_{t=1}^{T} (r_t - \widehat{\mu})^3}{\widehat{\sigma}^3}$	
Kurtosis:	$\widehat{\mathbf{K}} = \frac{\frac{1}{T} \sum_{t=1}^{T} (r_t - \widehat{\mu})^4}{\widehat{\sigma}^4}$	
Theoretical mome	nts for $r \sim N(0, 1)$	
Theoretical mome	nts for $r \sim N(0, 1)$	
Theoretical momen Expected value:	nts for $\boldsymbol{r} \sim \boldsymbol{N}(\boldsymbol{0}, \boldsymbol{1})$ $\mu = E(r_t) = 0$	
Theoretical momen Expected value: Variance:	hts for $r \sim N(0, 1)$ $\mu = E(r_t) = 0$ $\sigma^2 = E((r_t - \mu)^2) = 1$	
Theoretical momen Expected value: Variance: Standard deviation:	That s for $r \sim N(0, 1)$ $\mu = E(r_t) = 0$ $\sigma^2 = E((r_t - \mu)^2) = 1$ $\sigma = 1$	
Theoretical momen Expected value: Variance: Standard deviation: Skewness:	hts for $r \sim N(0, 1)$ $\mu = E(r_t) = 0$ $\sigma^2 = E((r_t - \mu)^2) = 1$ $\sigma = 1$ $S = E((r_t - \mu)^3) = 0$	
Theoretical moment Expected value: Variance: Standard deviation: Skewness: Kurtosis:	hts for $r \sim N(0, 1)$ $\mu = E(r_t) = 0$ $\sigma^2 = E((r_t - \mu)^2) = 1$ $\sigma = 1$ $S = E((r_t - \mu)^3) = 0$ $K = E((r_t - \mu)^4) = 3$	

Financial series characteristics

- 1. Fat tails
 - Kurtosis above 3
- 2. Asymmetry of ups and downs (deeper declines)
 - Negative skewness

Data for WIG returns

```
(daily data from 10.02.2016 - 11.02.2021)
```

$\hat{\mu} = 0.000217$	ightarrow annualized return 0.054
$\hat{\sigma} = 0.0115$	ightarrow annualized std. dev. 0.183
$\hat{S} = -1.45$	
$\widehat{K} = 21.33$	

Norte: Standard deviation vs synthetic risk index in SRRI in KIID (link)





v=Inf

v=5 v=3





$$Q_p = \mu + \sigma \left(T_v^{-1}(p) \sqrt{\frac{v-2}{v}} \right)$$

Important!!! Differences between R functions: rt/qt/dt/ct - stats package rdist/qdist/ddist/cdist - rugarch package

* More details in *Student t Distributed Linear Value-at-Risk* – <u>link</u>

Ŷ

4

-2

0

Qemp

2

4

Financial series characteristics

- 1. Fat tails
- Asymmetry of ups and downs (deeper declines) 2.
- No autocorrelation of returns 3.
 - $cor(r_t, r_{t-p}) = 0$
- Non-linear autocorrelation dependencies 4.
 - no autocorrelation \neq independence
 - $cor(r_t^2, r_{t-p}^2) \neq 0$: volatility clustering
 - $cor(r_t^2, r_{t-p}) \neq 0$: leverage effect
 - ale $cor(r_t, r_{t-p}^2) = 0$

Financial series characteristics: WIG







10

Lag

15





Ljunga-Boxa test (adjusted portmanteau)

Test for autocorrelation of order *H* with null:

$$H0: \rho_1 = \rho_2 = \dots = \rho_H = 0$$

Test statistics:

$$Q_{LB} = T(T+2) \sum_{h=1}^{H} \frac{\hat{\rho}_h^2}{T-h}$$

Under the null H0 statistics Q_{LB} is $\chi^2(H)$ distributed

Wyniki dla WIG: data: y0; LB = 45.034, df = 20, p-value = 0.001092 data: y0^2; LB = 1385.4, df = 20, p-value < 2.2e-16

Topic 2: Exercises

Exercise 2.1. Draw QQplot vs normal distribution for the below data:

0.49 -0.56 0.61 0.67 0.82 0.85 -2.04 -0.65 0.80 -1.00

knowing that the quantiles of normal distribution are:

q	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95
qnorm(q)	-1.64	-1.04	-0.67	-0.39	-0.13	0.13	0.39	0.67	1.04	1.64

Exercise 2.2. For selected investment fund returns:

a. Calculate: mean, std. dev., skewness and kurtosis (annualized)

- b. Verify if skewness is null and kurtosis equal to 3
- c. Standardize returns (R^*)
- d. Compare empirical density of R^* to the pdf of normal distribution

e. Draw QQ plot vs normal distribution

f. Estimate t-Student parameters (degree of freedom)

- g. Draw QQ plot vs t-Student pdf
- h. Plot ACF to visualize if:

 $cor(r_t, r_{t-p}) = 0; \ cor(r_t^2, r_{t-p}^2) \neq 0; \ cor(r_t^2, r_{t-p}) \neq 0 \text{ and } cor(r_t, r_{t-p}^2) = 0$

i. Check for autocorrelation of returns and their squares with the LB test

Topic 3

Risk measures: VaR and ES

- Value at Risk(VaR) and Expected Shortfall (ES) definitions
- Stages of VaR and ES calculation
- Metods of estimating VaR and ES
- Historical simulation
- Parametric models for VaR and ES
- Monte-Carlo simulation
- Cornish-Fisher expansion

Risk \neq **standard deviation**

Three series with E(Y) = 0 i Sd(Y) = 1 (Danielson, 2011)



Risk: Value at Risk and Expected Shortfall

Value at Risk, VaR:

Definition 1: $P(r \le VaR_p) = p$ Definition 2: $p = \int_{-\infty}^{VaR_p} f(r)dr$

Expected shortfall, ES:

Definition 1: $ES_p = E(r|r \le VaR_p)$ Definition 2: $ES_p = \frac{1}{p} \int_{-\infty}^{VaR_p} rf(r) dr$ Definition 3: $ES_p = \frac{1}{p} \int_{0}^{p} VaR_s ds$



Value at Risk and Expected Shortfall

Value at Risk and Expected Shortfall

VaR/ES calculation stages

- 1. Setting tolerance level: *p*
- 2. Setting horizon: *H*
- 3. Choosing estimation sample period 1: *T*
- 4. Choosing a model + backtesting method
- 5. VaR/ES computation (for period T + 1)
- **Basel ii/iii:** VaR as a Risk measure (<u>link</u>, p. 44)
- **Basel iv:** plans to chanes into ES (<u>link</u>, p. 52)

Value at Risk: Basel II

Quantitative standards Basel II

- a. 99th percentile VaR must be computed on a daily basis
- b. In calculating VaR the minimum "holding period" will be 10 trading days.
 Banks may use VaR numbers calculated according to shorter holding periods scaled up to ten days by the square root of time
- c. The choice of sample period for calculating VaR is constrained to a minimum length of one year.
- d. banks will be free to use models based, for example, on variance-covariance matrices, historical simulations, or Monte Carlo simulations
- e. The multiplication factor will be set by individual supervisory authorities on the basis of their assessment of the quality of the bank's risk management system, subject to an absolute minimum of 3. Banks will be required to add to this factor a "plus" directly related to the ex-post performance of the model, thereby introducing a builtin positive incentive to maintain the predictive quality of the model. The plus will range from 0 to 1 based on the outcome of so-called "backtesting."

Source: Basle Committee on Banking Supervision, 1996. AMENDMENT TO THE CAPITAL ACCORD TO INCORPORATE MARKET RISKS (<u>link</u>, s. 44)

VaR and ES calculation methods

- A. Parametric / non-parametric models
- B. Analytical formula / Monte-Carlo simulations
- C. Conditional / unconditional volatility

A. Non-parametric model: historical simulation

- We assume that the distribution of returns is well approximated by past/historical returns
- We sort past returns from the lowest to highest:

$$rs_1 < rs_2 < \cdots < rs_N$$

• VaR_p is equal to *p*-th quantile of distribution, so that for M = floor(pN)

$$VaR_p = rs_M$$

• ES_p is equal to the average of the worst returns lower than VaR_p

$$ES_p = \frac{1}{M} \sum_{1}^{M} rs_i$$

A. Non-parametric model: historical simulation for WIG



A. Non-parametric model: historical simulation

HS shortcomming: low precision of VaR, especially for low p!

Reason: we only use information about one quantile and not entire distribution Std. dev. for *p*-th quantile is : $S(VaR_p) = \sqrt{\frac{p(1-p)}{Tf^2}}$

For WIG:

$$p = 0.05;$$
 $N = 2587;$ $f = 4.98;$ $S(VaR_p) = 0.00086$

95% confidence interval:

$$P\{VaR_p \in (\widehat{VaR}_p - 1.96S(VaR_p); \widehat{VaR}_p + 1.96S(VaR_p))\} = 0.95$$

For WIG:

$$P(VaR_p \in (-0.0213; -0.0179)) = 0.95$$

A. Non-parametric model: historical simulation

Pros:

- Simplicity
- Easy to communicate
- No need to make assumptions
- Extension possibilities (e.g. for volatility clustering)

Cons:

- Full dependence on historical data
- Difficult to conduct counterfactual caluclations
- Low precision of VaR estimates

B. Parametric models

- We search for the distribution (pdf) of future returns: f(r)
- Knowing this distribution allows to calculate VaR and ES

$$p = \int_{-\infty}^{VaR_p} f(r) dr$$

$$ES_p = \frac{1}{p} \int_{-\infty}^{VaR_p} rf(r) dr$$

B. Parametric models: normal distribution

If
$$r \sim N(\mu, \sigma^2)$$
 then:

$$VaR_p = \mu + \sigma \Phi^{-1}(p)$$
$$ES_p = \mu + \sigma \frac{\phi(\Phi^{-1}(p))}{p}$$

where Φ i ϕ are pdf and cdf for N(0,1)

Tables for $r \sim N(0,1)$ are (with minus)

р	0.5	0.1	0.05	0.025	0.01	0.001
VaR	0	1.282	1.645	1.960	2.326	3.090
ES	0.798	1.755	2.063	2.338	2.665	3.367

Note: we assume $r_{T+1} \sim N(\mu, \sigma^2)$ and calculate VaR_{T+1} and ES_{T+1}





Normal distribution

C. Monte Carlo simulations

- Assume we know DGP but cannot derive analitical formula for VaR/ES
- We can resort to Monte Carlo simulations.
- MC steps:
 - 1. Create "N" artificial observations from known DGP: $r^{(n)}$ for n = 1, 2, ..., N
 - 2. Sort artifical returns from lowers to highest: $rs^{(1)} \le rs^{(2)} \le ...$
 - 3. Set M = floor(pN) and calculate:

$$VaR_p = rs^{(M)}$$
 and $ES_p = \frac{1}{M} \sum_{1}^{M} rs^{(i)}$

C. Monte Carlo simulations

MC vs analytical calculations comparison for WIG:

• metoda parametryczna vs. MC dla rozkładu normalnego ($N = 100\ 000$)

VaR	
Analytical method:	-0.02091533
MC simulations:	-0.02095668

ES:

Analytical method :	-0.02624805
MC simulations :	-0.02626429

Fat tails

Two methods to account for "fat tails":

- *t*-Student distribution
- Cornisha-Fisher expansion: correction of quantiles from normal distribution for skewness and kurtosis

More sophisticated methods (beyond this course):

• EVT, extreme value theory

Fat tails: t-Student distribution

Reminder:

Variance t_v :

Quantile *p*:

$$Var(t_{v}) = \frac{v}{v-2}$$
$$Q_{p} = \mu + \sigma \left(T_{v}^{-1}(p) \sqrt{\frac{v-2}{v}} \right)$$

VaR:

$$VaR_p = \mu + \sigma\left(T_v^{-1}(p)\sqrt{\frac{v-2}{v}}\right)$$

ES, numerical integration:

$$ES_p = \mu + \sigma \frac{1}{p} \int_0^p \left(T_v^{-1}(s) \sqrt{\frac{v-2}{v}} \right) ds = \frac{1}{p} \int_0^p VaR_s \, ds$$

Fat tails: t-Student distribution





Fat tails: Cornish-Fisher expansion

 Cornish-Fisher expansion accounts for skewness and kurtosis (also higher moments*) in quantile calculations:

$$VaR_{p} = \mu + \sigma \left(\gamma_{p} + \frac{\gamma_{p}^{2} - 1}{6}S + \frac{\gamma_{p}^{3} - 3\gamma_{p}}{24}(K - 3) - \frac{2\gamma_{p}^{3} - 5\gamma_{p}}{36}S^{2} \right)$$

where $\gamma_{p} = \Phi^{-1}(p)$.

• For normal distribution (S = 0 and K = 3), the formula simplifies to:

$$VaR_p = \mu + \sigma \gamma_p$$

* More on Cornish-Fisher expansion – link

** MRM methodology - <u>link</u>

Fat tails: Cornish-Fisher expansion WIG example

S = -0.450K = 7.055 $\gamma_p = -1.645$

$$\left(\gamma_p + \frac{\gamma_p^2 - 1}{6}S + \frac{\gamma_p^3 - 3\gamma_p}{24}(K - 3) - \frac{2\gamma_p^3 - 5\gamma_p}{36}S^2\right) = -1.687$$

Models comparison



Tolerance level and VaR

Topic 3: Exercises

Exercise 3.1. The distribution of log-returns are t-Student with 5 degrees of freedom. Compute VaR for a selected tolerance level p if expected return is 0.5% and std. dev. amounts to 6%. Critical values for t-Student with v = 5 are equal to:

1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
-3.365	-2.757	-2.422	-2.191	-2.015	-1.873	-1.753	-1.649	-1.558	-1.476

Note: critical values were generated using function qt (p, 5)

Exercise 3.2. We know that returns are uniformly distributed over the interval (-0.01;0.01), $r \sim U(-0.01,0.01)$. Calculate VaR and ES for p=0.05 and p=0.10.

Exercise 3.3. Calculate VaR using Cornish-Fisher expansion if returns moments are as follows: $\mu = 0.5\%$, $\sigma = 5\%$, S = -1, K = 7. Assume the tolerance level at p = 0.05 or 0.025. $[\Phi^{-1}(0.05) = -1.645$ and $\Phi^{-1}(0.025) = -1.960$]

Exercise 3.4*. Create a function in R, which will allow you to compute ES consistent with the Cornish-Fisher expansion. Use the function to compute ES for p = 0.05 or 0.025 and moments from exercise 3.3.
Temat 3: Exercises

Exercise 3.5.

For return of the selected investment fund (from Exercise 2.2), do the following::

- a. Consider which of the four methods discussed so far (HS, normal, t-Student, CF) you think is appropriate for VaR calculation
- b. Calculate the VaR and ES values on the basis of the above 4 methods for a tolerance level of 5%. Why do the results differ?
- c. Create a plot for the empirical density function, the density of the normal and t-Student distribution. Plot the values from point b on the graph.
- d. Calculate the VaR and ES values on the basis of the above 4 methods for a tolerance level of 1% and compare tchem with values from point b.
- e. Discuss the obtained results

Topic 4

Volatility clustering

- Volatility clustering
- Moving average (MA)
- Exponentially Weighted Moving Average (EWMA)
- GARCH model

Financial series characteristics

- 1. Fat tails
- 2. Asymmetry of ups and downs (deeper declines)
- 3. No autocorrelation of returns
 - $cor(r_t, r_{t-p}) = 0$
- 4. Non-linear autocorrelation dependencies
 - $cor(r_t^2, r_{t-p}^2) \neq 0$: volatility clustering
 - $cor(r_t^2, r_{t-p}) \neq 0$: leverage effect

Volatility clustering and VaR/ES from t-Student distribution



5% VaR and ES from unconditional variance model

Methods of volatility modelling

- MA: Moving Average
- EWMA: Exponentially Weighted Moving Average
- GARCH model: Generalized Autoregressive Conditional Heteroskedasticity
- SV: Stochastic Volatility
- IV: Implied Volatility

Important:

$$VaR_{T+1|T} = VaR(\mu_{T+1|T}, \sigma_{T+1|T}, \dots)$$

hence we need to calculate a forecast $\sigma_{T+1|T}$

A. Moving average

Formula for variance forecast (*T* – moment forecast formulation):

$$\sigma_{T+1}^2 = \frac{1}{W} \sum_{s=0}^{W-1} (r_{T-s} - \mu)^2$$

- Note 1: the value depends on window length W
- Note 2: we use information up to moment T
- Note 3: the above formula can be written down as weighted average with equal weights

$$\sigma_{T+1}^2 = \sum_{s=0}^{W-1} w_s (r_{T-s} - \mu)^2$$
, where $w_s = \frac{1}{W}$

A. VaR and ES from moving average model



5% VaR and ES from MA model

B. Exponentially Weighted Moving Average, EWMA

Variance forecast calculated as a weighted average of past observations:

$$\sigma_{T+1}^2 = \sum_{s=0}^{\infty} w_s (r_{T-s} - \mu)^2$$

in which weights form a geometric sequance:

$$w_s = \lambda w_{s-1} \qquad \leftrightarrow \qquad w_s = \lambda^s \times \frac{1-\lambda}{\lambda}$$

Note: given that weights sum to unity, this implies that $w_0 = 1 - \lambda$

$$\sigma_{T+1}^2 = (1-\lambda)(r_T - \mu)^2 + \lambda \sigma_{t-1}^2$$

In RiskMetrics (JP Morgan, <u>link</u>) parameters λ and μ are not estimated but calibrated. For daily data the proposed values are:

$$\lambda = 0.94$$
 and $\mu = 0$

VaR and ES from EWMA

• EWMA model:

$$\begin{aligned} r_t &\sim D(0, \sigma_t^2) \\ \sigma_t^2 &= (1 - \lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2 \end{aligned}$$

Variance forecast:

$$\sigma_{T+1}^2 = (1-\lambda)r_T^2 + \lambda\sigma_T^2$$

Let F_D be the cdf of D distribution, which implies that:

$$VaR_{p,T+1} = \sigma_{T+1}F_D^{-1}(p)$$

ES amounts to:

$$ES_{p,T+1} = \sigma_{T+1} \frac{1}{p} \int_0^p F_D^{-1}(s) ds$$

VaR and ES from EWMA





Index

GARCH as EWMA extension

RiskMetrics:

$$r_t \sim D(0, \sigma_t^2)$$

$$\sigma_t^2 = (1 - \lambda)r_{t-1}^2 + \lambda \sigma_{t-1}^2$$

• GARCH(1,1):

$$\begin{split} r_t &= \mu + \epsilon_t, \qquad \epsilon_t \sim D(0, \sigma_t^2) \\ \sigma_t^2 &= \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2 \end{split}$$

• EWMA restrictions (EWMA=Integrated GARCH, IGARCH):

$$\mu = 0$$
; $\omega = 0$; $\alpha = 1 - \lambda$; $\beta = \lambda$

C. GARCH models

Benchmark GARCH(1,1) model specification:

$$\begin{aligned} r_t &= \mu + \epsilon_t, \qquad \epsilon_t \sim D(0, \sigma_t^2) \\ \sigma_t^2 &= \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned}$$

where $\omega > 0$ and $\alpha, \beta \ge 0$.

• Equilibrium (unconditional) variance is:

$$\bar{\sigma}^2 = \frac{\omega}{1 - (\alpha + \beta)}$$

- Extensions:
 - Leverage effect: EGARCH, GJR-GRACH
 - Risk premium: GARCH-in-Mean

VaR and ES from GARCH

GARCH model:

$$\begin{aligned} r_t &= \mu + \epsilon_t, \qquad \epsilon_t \sim D(0, \sigma_t^2) \\ \sigma_t^2 &= \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned}$$

Variance forecast:

$$\sigma_{T+1}^2 = \omega + \alpha \epsilon_T^2 + \lambda \sigma_T^2$$

• Let *F*_D be the cdf of *D* distribution, which implies that:

$$VaR_{p,T+1} = \mu + \sigma_{T+1}F_D^{-1}(p)$$

ES amounts to:

$$ES_{p,T+1} = \mu + \sigma_{T+1} \frac{1}{p} \int_0^p F_D^{-1}(s) ds$$

C. VaR and ES from GARCH model

5% VaR and ES from GARCH(1,1))





Veryfying GARCH models

• To veryfy the quality of GARCH model we test standarized residuals:

$$u_t = \frac{\epsilon_t}{\sigma_t}$$

- The should be characterized by:
 - no autocorrelation
 - no autocorrelation of squares
 - QQ plot should indicate that the assumed distribution is correct

$$u_t \sim IID D(0,1)$$

Topic 4: Exercises

Exercise 4.1. The model for the rate of return (expressed as %) is:

$$r_t = 0.1 + \epsilon_t, \qquad \epsilon_t \sim N(0, \sigma_t^2)$$

$$\sigma_t^2 = 0.4 + 0.1\epsilon_{t-1}^2 + 0.8\sigma_{t-1}^2$$

It is known that $r_T = -0.9$ and $\sigma_T^2 = 4$.

- 1. Calculate a forecast for the moments μ_{T+1} and σ_{T+1}
- 2. Copute VaR_{T+1} and ES_{T+1} for toletance level p = 5%
- 3. What is the equilibrium variance in this model?

Exercise 4.2. For your chosen asset:

- 1. Compute variance forecast σ_{T+1}^2 using discussed methods (constant variance, MA, EWMA, GARCH).
- 2. Assume t-Student distribution with v = 5 and compute VaR/ES for the above methods (for tolerance level p = 1% and p = 5%)
- 3. Repeat points 1 and 2 for notmal distribution
- 4. Create a table with the results

Topics 1-4 presentation

Content of the presentation:

- a. <1.0p> Information about the fund (KIID), including fees
- b. <1.5p> Historical data + returns characteristics (moments, QQ plot, density plot)
- c. <1.5p> GARCH model estimates (+ selected plots)
- d. <3.0p> VaR and ES (1% i 5%) calculated with:
 - Historical simulation
 - Parametric method (normal / t-Student)
 - Cornish-Fisher expansion
 - EWMA
 - GARCH

Note: all results should be presented in one table.

e. <1.0p> A plot: VaR vs tolerance level for 5 above methods

f. <1.0p> General discussion about the risk of investing in a given fund

Additionally, 1 p. for the quality of presentation and the speech. Time limit: 5 minutw. Avoid a large number of slides (7 slides is a good choice). Presentation in pdf file entitled *SurnameName.pdf* download to MT.

Topic 5

VaR and ES for longer horizons

- Ssquare root of time method
- Cornish-Fisher expansion for $H{>}1$
- Monte Carlo simulations
- Bootstrap
- *H*-period returns

VaR/ES for longer horizons

- § So far, we have learnt methods of computing VaR and ES for shortest horizons, ie. one step ahead (H=1)
- § In the decision making, we quite often need the information on investment risk for longer horizons (a week, a month, a year, 5 years). In such cases, we need to compute VaR/ES for the variable $y_H = \sum_{h=1}^{H} r_h$.
- § There are two approaches:
 - 1. analytic (eg. square root of time)
 - 2. numerical simulation (Monte Carlo, bootstraping)
- § For very long horizons (>1 month), it is recommended to supplement VaR estimates with scenario analyses (next topic stress tests).

A. Analytic methods

Expected value and variance for longer horizons

Assume, that the expected value and variance of returns r_t are:

Expected value:	μ	$= E(r_t)$
Variance:	σ^2	$= E \big((r_t - \mu)^2 \big)$
Standard deviation:	σ	$=\sqrt{\sigma^2}$

If r_h are IID (independently and identically distributed), then for the cumulative returns $y_H = \sum_{h=1}^{H} r_h$ we have:

Expected value: $\mu_H = H\mu$ Variance: $\sigma_H^2 = H\sigma^2$ Standard deviation: $\sigma_H = \sqrt{H}\sigma$

A. Analytic methods

Normal distribution: square root of time

§ If $r \sim N(\mu, \sigma^2)$, then for one step horizon:

 $VaR = \mu + \sigma \Phi^{-1}(p)$ oraz $ES = \mu - \sigma \frac{\phi(\Phi^{-1}(p))}{p}$

with Φ and ϕ are the cumulative distribution function (cdf) and the probability density function (pdf) of the standard normal distribution N(0,1).

§ Since
$$y_H = \sum_{h=1}^{H} r_h \sim N(H\mu, H\sigma^2)$$
 then:
 $VaR_H = H\mu + \sqrt{H}\sigma\Phi^{-1}(p)$ and $ES_H = H\mu - \sqrt{H}\sigma\frac{\phi(\Phi^{-1}(p))}{p}$

§ Assuming
$$\mu = 0$$
, we get:
 $VaR_H = \sqrt{H}VaR$ and $ES_H = \sqrt{H}ES$

Thus, we call this the square root of time method.

A. Analytic methods SRT in Basel II

Quantitative standards Basel II

- a. 99th percentile VaR must be computed on a daily basis
- b. In calculating VaR the minimum *"holding period"* will be 10 trading days. Banks may use VaR numbers calculated according to shorter holding periods scaled up to ten days by the square root of time
- c. The choice of sample period for calculating VaR is constrained to a minimum length of one year.
- d. banks will be free to use models based, for example, on variance-covariance matrices, historical simulations, or Monte Carlo simulations
- e. The multiplication factor will be set by individual supervisory authorities on the basis of their assessment of the quality of the bank's risk management system, subject to an absolute minimum of 3. Banks will be required to add to this factor a "plus" directly related to the ex-post performance of the model, thereby introducing a built in positive incentive to maintain the predictive quality of the model. The plus will range from 0 to 1 based on the outcome of so-called "backtesting."

Source: Basle Committee on Banking Supervision, 1996. AMENDMENT TO THE CAPITAL ACCORD TO INCORPORATE MARKET RISKS (<u>link</u>, s. 44)

A. Analytic methods

Skewness and kurtosis for longer horizons

§ Higher moments for $r_t \sim D(\mu_I \sigma^2)$:

Skewness:
$$S = M_3 / \sigma^3$$
 with $M_3 = E((r_t - \mu)^3)$
Kurtosis: $K = M_4 / \sigma^4 - 3$ with $M_4 = E((r_t - \mu)^4)$

§ If r_h are IID, the for the variable $y_H = \sum_{h=1}^{H} r_h$ the moments are:

Skewness:	S_H	$= S I \sqrt{H}$
Kurtosis:	K_H	= K / H

A. Analytic methods

Cornish-Fischer formula for longer horizons

Cornish-Fisher formula for
$$H = 1$$
:

$$VaR = \mu + \sigma \left(\gamma_p + \frac{\gamma_p^2 - \mathbf{1}}{\mathbf{6}} S + \frac{\gamma_p^3 - \mathbf{3}\gamma_p}{\mathbf{24}} K - \frac{\mathbf{2}\gamma_p^3 - \mathbf{5}\gamma_p}{\mathbf{36}} S^2 \right)$$
with $\gamma_p = \Phi^{-1}(p)$

Cornish-Fisher formula for general H:

$$VaR_{H} = \mu H + \sigma \sqrt{H} \left(\gamma_{p} + \frac{\gamma_{p}^{2} - 1}{6} \frac{S}{\sqrt{H}} + \frac{\gamma_{p}^{3} - 3\gamma_{p}}{24} \frac{K}{H} - \frac{2\gamma_{p}^{3} - 5\gamma_{p}}{36} \frac{S^{2}}{H} \right)$$

Cornish-Fisher formula in European Commission regulation: Regulatory Technical Standards (RTS) for packaged retail and insurance-based investment products (PRIIPs) - <u>link</u>

$$\mathrm{Exp}\left[M1^*N + \sigma \sqrt{N}^* \left(-1{,}28 + 0{,}107^*\mu_1/\sqrt{N} + 0{,}0724^*\mu_2/N - 0{,}0611^*\mu_1^2/N\right) - 0{,}5\,\sigma^2N\right]$$

A. Analytic methods Results for WIG index



- B. Numerical methods Monte Carlo simulations
- § Assume, that we know the data generating process, DGP
- § MC steps for computing VaR/ES for general H:
 - 1. Generate return path r_1, r_2, \dots, r_H for horizon H
 - 2. Compute cumulated returns $y_H = \sum_{h=1}^{H} r_h$
 - 3. Repeat (1)-(2) *N* times and store $y_{H}^{(n)}$ for n = 1,2,...,N
 - 4. Sort increasingly cumulated returns

$$ys_{H}^{(1)} \leq ys_{H}^{(2)} \leq \dots$$

- 1. Let M = floor(pN)
- 2. Output:

$$VaR_{H} = ys_{H}^{(M)}$$
 and $ES_{H} = \frac{1}{M}\sum_{i=1}^{M} ys_{H}^{(i)}$

B. Numerical methods

Monte Carlo simulations for normal distribution

§ Assume $r \sim N(\mu, \sigma^2)$

- § MC steps for computing VaR/ES for general H:
 - 1. Generate *H* returns $r_1, r_2, ..., r_H$ from $N(\mu, \sigma^2)$
 - 2. Compute cumulated returns $y_H = \sum_{h=1}^{H} r_h$
 - 3. Repeat (1)-(2) *N* times and store $y_{H}^{(n)}$ for n = 1, 2, ..., N
 - 4. Sort increasingly cumulated returns

$$ys_{H}^{(1)} \leq ys_{H}^{(2)} \leq \dots$$

- 1. Let M = floor(pN)
- 2. Output:

$$VaR_{H} = ys_{H}^{(M)}$$
 and $ES_{H} = \frac{1}{M}\sum_{i=1}^{M} ys_{H}^{(i)}$

B. Numerical methods

Monte Carlo simulations for t-Student distribution

- § Assume $r \sim t_v (\mu_{\sigma} \sigma^2)$
- § MC steps for computing VaR/ES for general H:
 - 1. Generate *H* values t_1, t_2, \dots, t_H from t_v distribution
 - 2. Generate returns r_1, r_2, \dots, r_H from the formula $r_h = \mu + \sigma \times t_h \sqrt{\frac{v-2}{v}}$
 - 3. Compute cumulated returns $y_H = \sum_{h=1}^{H} r_h$
 - 4. Repeat (1)-(3) N times and store $y_H^{(n)}$ for n = 1,2,...,N
 - 5. Sort increasingly cumulated returns

$$ys_{H}^{(1)} \leq ys_{H}^{(2)} \leq \dots$$

- 1. Let M = floor(pN)
- 2. Output:

$$VaR_{H} = ys_{H}^{(M)}$$
 and $ES_{H} = \frac{1}{M}\sum_{i=1}^{M} ys_{H}^{(i)}$

B. Numerical methods

Monte Carlo simulations for GARCH model

- § Assume $r_t \sim \text{GARCH}$
- § MC steps for computing VaR/ES for general H:
 - 1. Estimate GARCH model parameters
 - 2. Simulate *H*-return path r_1, r_2, \dots, r_H , conditionally on the last in-sample observation
 - 3. Compute cumulated returns $y_H = \sum_{h=1}^{H} r_h$
 - 4. Repeat (2)-(3) *N* times and store $y_{H}^{(n)}$ for n = 1, 2, ..., N
 - 5. Sort increasingly cumulated returns

$$ys_{H}^{(1)} \le ys_{H}^{(2)} \le \dots$$

- 1. Let M = floor(pN)
- 2. Output:

$$VaR_{H} = ys_{H}^{(M)}$$
 and $ES_{H} = \frac{1}{M}\sum_{i=1}^{M} ys_{H}^{(i)}$

A. Numerical methods Monte Carlo simulations



C. Numerical methods

Bootstrap - historical simulations for IID returns

Assuming IID returns, we can use historical simulation

- § Bootstrap steps for computing VaR/ES for general *H*:
 - 1. Select randomly with replacements H returns $r_1, r_2, ..., r_H$ from historical sample $r_{1:T}$
 - 2. Compute cumulated returns $y_H = \sum_{h=1}^{H} r_h$
 - 3. Repeat (1)-(2) *N* times and store $y_{H}^{(n)}$ for n = 1, 2, ..., N
 - 4. Sort increasingly cumulated returns

$$ys_H^{(1)} \leq ys_H^{(2)} \leq \dots$$

- 1. Let M = floor(pN)
- 2. Output:

$$VaR_H = ys_H^{(M)}$$
 and $ES_H = \frac{1}{M} \sum_{1}^{M} ys_H^{(i)}$

What if returns are autocorrelated?

Method 1. Try to express DGP using a suitable model (eg. GARCH, ARMA) and use MC to simulate from that model.

Method 2. Use *H*-step returns $r_{t,H} = \ln P_t - \ln P_{t-H}$

and perform computations as for one step ahead VaR.

Warning: method 2 significantly reduces the sample size to **floor**(T/H), which makes the backtesting quite problematic or even infeasible.

Model comparison

	VaR ₅	ES_5
rozk. norm.	-0,0323	-0,0419
Cornish-Fischer	-0,0335	
rozk. norm. MC	-0,0330	-0,0435
rozklad t MC	-0,0312	-0,0458
GARCH MC	-0,0312	-0,0403
HS, bootstrap	-0,0318	-0,0439
rozkl. norm. (H-okresowe stopy)	-0,0327	-0,0410

Topic 5: Exercises

Exercise 5.1. Log-returns for an asset are $N(1; 2^2)$ distributed. Compute VaR and ES for H = 1, 4, 9 and tolerance levels p = 1% and p = 5%. VaR and ES for N(0,1) distribution are

р	0.5	0.1	0.05	0.025	0.01	0.001
VaR	0	1.282	1.645	1.960	2.326	3.090
ES	0.798	1.755	2.063	2.338	2.665	3.367

Exercise 5.2. Compute VaR for H = 4 using Cornish-Fischer formula for an asset, whose returns have the following characteristics: $\mu = 0.5\%$, $\sigma = 5\%$, S = -1, K = 7. Assume tolerance levels p = 0.05 and 0.025. $[\Phi^{-1}(0.05) = -1.645$ and $\Phi^{-1}(0.025) = -1.960$]

Topic 5: Exercises

Exercise 5.3. Using the share prices of a selected investment fund, compute VaR and ES for H = 10, with 6 methods discussed in class (normal, CF, t, GARCH, boot, normal *H*-step) for tolerance 5%. Are there any differences? Repeat your computations for tolerance 1%.

Exercise 5.4* Create R scripts computing VaR and ES for EWMA model with normal distribution for general horizon H. Compute VaR and ES for H = 10 and compare with the results from the GARCH model (Ex. 5.3).

Exercise 5.5* Create R scripts computing VaR and ES for EWMA model with historical distribution for general horizon H. Compute VaR and ES for H = 10 and compare with the results from the GARCH model (Ex. 5.3) and the EWMA-norm model (Ex. 5.4)

Topic 6

Stress tests

- Stress test and VaR/ES
- Sensitivity analysis
- Scenario analysis
- Historical and hypothetical scenarios
- Stressed-VaR

Stress tests

Stress tests evaluate the influence of low probability events with potentially high negative impact on the portfolio value (or the financial standing of a company, the stability of a financial system, etc.).

Examples:

- § stock market crash
- § currency devaluation
- § liquidity loss
- § default of a debtor
- § loss of an important client



In the automotive industry, stress tests correspond to crash tests.

Stress tests

Value at risk / expected shortfall:

- § normal market condition
- § short time horizon
- § statistical approach

Stress tests:

- § atypical/crisis market conditon
- § longer horizon
- § scenario approach

Important:

Value at risk and stress tests are complementary risk measures.

Stress tests: scheme of analysis

Stage 1. Sensitivity analysis

reaction of portfolio to changes in risk factors, eg.

- § stock indices,
- § yield curves,
- § exchange rates,
- § commodity prices.

Stage 2. Scenario analysis evaluates the change of portfolio, given various changes in market conditions, e.g.

- § credit crunch,
- § default of the main client,
- § intensified terrorist attacks,
- § pandemic

Stage 3. Stress tests

stress condition = worst possible scenario

Stress tests: what scenarios?

- 1. Historical scenarios, e.g.
 - § Great depression from 1930s
 - § ERM crisis from 1992
 - § Asian crisis from 1997
 - § Financial crisis from 2007-2009



Jul 9, 2007:	67 772,91
Feb 18, 2009:	20 370,29
70% decline in 1.5 years	

 EUR/PLN exchange rate:

 Jul 31, 2008:
 3,20 PLN/EUR

 Feb 18, 2009:
 4,90 PLN/EUR

 35% depreciation in 0.5 year

§ COVID-19 pandemic



Stress tests: what scenarios?

2. Hypothetical scenarios, ie. events that not necessarily occurred in the past but may possibly happen in the future

- § sudden climate changes
- § sovereign defaults
- § new regulations
- § Polexit
- § war on Korean peninsula



Stress tests: what scenarios?

3. "Standard" hypothetical scenarios

- eg. proposed by the <u>Derivatives Policy Group (1995)</u>:
 - § parallel yield curve shifts of 100 basis points up and down
 - § steepening and flattening of the yield curves (2's to 10's) by 25 basis points;
 - § increase and decrease in equity index values by 10 percent
 - § increase and decrease in the exchange value of foreign currencies by
 6 percent (major currencies) and 20 percent (other currencies)
 - § increase and decrease in swap spreads by 20 basis points.

Stress tests: example

Assumed portfolio structure:

- A1, 40%: domestic government bonds, duration 5 years
- A2, 10%: domestic corporate bonds, duration 3 years
- A3, 30%: domestic stocks
- A4, 20%: foreign stocks

Stage 1. Sensitivity analysis:

	A1	A2	A3	A4	Portfolio
RF1: 1% increase of stock indices (domestic and foreign)	0%	0%	1%	1%	0,5%
RF2: uniform shift (increase) of the domestic yield curve by 100 basis points (bp)	-5%	-3%	0%	0%	-2,3%
RF3: domestic currency depreciation by 1%	0%	0%	0%	1%	0,2%
RF4: increase of the corporate debt spread by 100 bp	0%	-3%	0%	0%	-0,3%
RF5: increase of commodity prices by 1%	0%	0%	0%	0%	0,0%

Note: RF - Risk factor, A - asset

Stress tests: example

	Portfolio
RF1: 1% increase of stock indices (domestic and foreign)	0,5%
RF2: uniform shift (increase) of the domestic yield curve by 100 bp	-2,3%
RF3: domestic currency depreciation by 1%	0,2%
RF4: increase of the corporate debt spread by 100 bp	-0,3%
RF5: increase of commodity prices by 1%	0,0%

Sensitivity analysis

		Portfel
Scenario analysis	S1: decrease/increase of stock indices (domestic and foreign) by 10%	±5,0%
	S2: uniform shift (increase/decrease) of the domestic yield curve by 100 bp	±2,3%
	S3: domestic currency appreciation/depreciation by 20%	±4,0%
	S4: : decrease/increase of the corporate debt spread by 100 pb.	±0,3%

Portfolio value for the worst case scenario:

decrease of stock indices, increase of interest rates, currency appreciation and increase of spread $\Delta \ln(P) = -5\% - 2,3\% - 4,0\% - 0,3\% = -11,6\%$

Stress tests: what scenarios?

We can assume a hypothetical return distribution usually higher standard deviation, change in correlations and also in higher moments

For instance, among "standard" hypothetical scenarios proposed by the <u>Derivatives Policy Group (1995)</u>, there are:

- § increase and decrease in all 3-month yield volatilities by 20 percent,
- § increase and decrease in equity index volatilities by 20 percent,
- § increase and decrease in foreign exchange rate volatilities by 20 percent.

In case of historical scenarios, we select periods with high volatility, high correlations or rapid declines.

Stressed VaR, S-VAR

Stressed value at risk is computed similarly to the basis value at risk, although with different more conservative assumptions on the return distribution (lower expected return, higher volatility, etc.).

Example. Stressed scenario in:

Regulation (EU) No 1286/2014 of the European Parliament and of the Council on key information documents for packaged retail and insurance-based investment products (PRIIPs), Annex V (link)

$$Scenario_{Stress} = e\left[{}^{w}\sigma_{S}*\sqrt{N}*\left(z_{\alpha} + \left[\frac{\left(Z_{\alpha}^{2}-1\right)}{6}\right]*\frac{\mu_{1}}{\sqrt{N}} + \left[\frac{\left(z_{\alpha}^{3}-3z_{\alpha}\right)}{24}\right]*\frac{\mu_{2}}{N} - \left[\frac{\left(2z_{\alpha}^{3}-5z_{\alpha}\right)}{36}\right]*\frac{\mu_{1}^{2}}{N}\right) - 0.5 \ ^{W}\sigma_{S}^{2}N\right]$$

It uses Cornish-Fischer formula with

- zero expected return $\mu = \mathbf{0}$,
- w_{σ_S} being the 99th percentile of rolling std deviation on 21-day windows,
- z_{α} being 1st percentile of standard normal distribution.

Stressed VaR, S-VAR

Example. Stressed scenario in:

Regulation (EU) No 1286/2014 of the European Parliament and of the Council on key information documents for packaged retail and insurance-based investment products (PRIIPs), Annex V (link)

$$\mathrm{Scenario}_{\mathrm{Stress}} = e\left[{}^{w}\sigma_{\mathrm{S}}*\sqrt{N}*\left(z_{\alpha} + \left[\frac{\left(Z_{\alpha}^{2} - 1\right)}{6}\right]*\frac{\mu_{1}}{\sqrt{N}} + \left[\frac{\left(z_{\alpha}^{3} - 3z_{\alpha}\right)}{24}\right]*\frac{\mu_{2}}{N} - \left[\frac{\left(2z_{\alpha}^{3} - 5z_{\alpha}\right)}{36}\right]*\frac{\mu_{1}^{2}}{N}\right) - 0.5 \ ^{W}\sigma_{\mathrm{S}}^{2}N\right]$$

It uses Cornish-Fischer formula with

- zero expected return $\mu = 0$,
- \mathbf{w}_{σ_S} being the 99th percentile of rolling std deviation on 21-day windows,
- z_{α} being 1st percentile of standard normal distribution.

In comparison to Cornish-Fisher for *H*-step horizon VaR:

$$VaR_{H} = \mu H + \sigma \sqrt{H} \left(\gamma_{p} + \frac{\gamma_{p}^{2} - \mathbf{1}}{\mathbf{6}} \frac{S}{\sqrt{H}} + \frac{\gamma_{p}^{3} - \mathbf{3}\gamma_{p}}{\mathbf{24}} \frac{K}{H} - \frac{\mathbf{2}\gamma_{p}^{3} - \mathbf{5}\gamma_{p}}{\mathbf{36}} \frac{S^{2}}{H} \right)$$

we observe, that in the stressed scenario we change the standard deviation and assume zero expected return.

Topic 6. Exercises

Exercise 6.1. For a selected investment fund, determine (qualitatively) the results of stress tests for standard scenarios

- § upward shift of the yield curve by 100 pb,
- § increase of stock prices by 10%,
- § depreciation of PLN by 20%,
- § increase of credit risk by 100 pb.

Exercise 6.2. For a selected investment fund, construct a historical scenario of the worst case loss in the one year horizon.

Exercise 6.3. An investment fund portfolio (PLN denominated) contains 2 asset classes: B1 – Polish corporate bonds, duration 2 years, B2 – German government bonds, duration 5 years.

- § Preform the sensitivity analysis of portfolio components to the following risk factors:
 - RF1 depreciation of PLN relative to EUR by 1%,
 - RF2 global upward shift of yield curves by 100pb,
 - RF3 increase of corporate bond spread by 100pb.
- § Assume the scenario with:
 - depreciation of PLN relative to EUR by 15%,
 - global increase of interest rates by 150pb,
 - increase of the corporate bond risk and their spread by 200pb.
 - Estimate the change in portfolio value under this scenario if it contains 60% and 40% of B1 and B2.

Topic 6. Exercises

Exercise 6.4. For the time series of a selected investment fund returns, compute 1 year horizon VaR (H=250) assuming the normal distribution of returns. Assume tolerance p = 1% and the square root of time method.

In a stress scenario, assume that μ^* is the minimal expected value and σ^* is the maximal standard deviation on 3-month rolling windows.

Exercise 6.5. For the time series of a selected investment fund returns, compute 1 year horizon VaR (H=250) with tolerance p = 1% using Cornish-Fisher formula. Assume a stress scenario with $\mu^* = 0$, and σ^* being 99th percentile of rolling standard deviations over 21-day windows. Compute S-VaR under these assumptions.

Topic 7

Backtesting

- Backtesting procedure
- VaR violations and tolerance level
- Binomial distribution
- Traffic lights method
- Kupiec test
- Christoffersen tests
- Tests power
- McNeil and Frey test for ES

Backtesting - basics

How to evaluate VaR/ES estimation methods?

- § In-sample model fit is not the best way to assess the quality of a risk model
- § VaR/ES is about the future returns, thus we want a good forecasting model
- § Backtesting tests the model accuracy in a simulated real-time scenario based on historical data
- § We assess the out-of-sample quality of VaR/ES predictions



sample 1 2 3 4 $T^* - 1$ $T^* + 1 T^* + 2T^* + 3$ T - 2 T - 1 T test * * * T - 2 T - 1 T 1 $T^* + 1 T^* + 1 T^* + 2 T^* + 3$ T - 2 T - 1 T 1 T - 2 T - 1 T T = 2 T - 1 T T = 2 T - 1 T T = 2 T - 1 T T = 2 T - 1 T T = 2 T - 1 T T = 2 T - 1 T T = 2 T - 1 T T = 2 T - 1 T T = 2 T - 1 T T = 2 T - 1 T T = 2 T - 1 T T = 2 T - 1 T T = 2 T - 1 T

We decide on:

- § the splitting point of the sample into estimation and evaluation subsamples (T^*)
- § tolerance (p) and horizon (H)

§ backtesting scheme: rolling vs recursive estimation windows We obtain time series (of size $n = T - T^*$):

- 1. predictions:
 $VaR_{t+1}, t = T^*, ..., T 1$

 2. realized values:
 $r_{t+1}, t = T^* + 1, ..., T 1$
- 3. VaR violations (excedances): $\eta_{t+1} = \begin{cases} \mathbf{1}, \text{ if } r_{t+1} \leq \mathbf{VaR}_{t+1} \\ \mathbf{0}, \text{ if } r_{t+1} > \mathbf{VaR}_{t+1} \end{cases}$

VaR violations

For the violation series:

$$\eta_{t+1} = \begin{cases} \mathbf{1}, & \text{ if } r_{t+1} \leq \text{VaR}_{t+1} \\ \mathbf{0}, & \text{ if } r_{t+1} > \text{VaR}_{t+1} \end{cases}$$

we derive:

§ number of violations:

$$n_1 = \sum_{t=T^*}^{T-1} \eta_{t+1}$$

§ number of nonviolations:

$$n_0 = n - n_1$$

§ empirical violation ratio:

$$\pi = \frac{n_1}{n} = \frac{n_1}{n_0 + n_1}$$

§ We expect $\pi \approx p!$

VaR violations - illustration



Basel II: traffic light approach

According to Basel II regulations, the required market risk capital reserve for a day *t* is

$\Xi_t \max \{ VaR_{t+10}, \overline{VaR}_t \},$

where:

- § VaR_{t+10} is the 10 business day horizon VaR with tolerance 1%,
- § \overline{VaR}_t is the mean of previous 60 values of VaR_{t+10}
- § Ξ_t depends on the numer of violations n_1 in previous 250 days

$$\Xi_t = \begin{cases} \mathbf{4}, & \text{if } \mathbf{10} \le n_1, \\ \mathbf{3} + \varphi(\mathbf{n}_1) & \text{if } \mathbf{5} \le n_1 \le \mathbf{9}, \\ \mathbf{3}, & \text{if } n_1 \le \mathbf{4}, \end{cases}$$

§ What determines Ξ_t ?

Distribution of the number of violations

§ We expect violations to be independent and to follow the Bernoulli distribution:

 $P(\eta_t = 1) = p_t$ $P(\eta_t = 0) = 1 - p_t$

- § Thus, the number of violations n_1 follows the binomial distribution B(n,p) with parameters n and p.
- What values of n₁ support the rejection of the model for n =
 250 and p = 5%? What if p = 1%?

Distribution of n_1 for n = 250

	p = 5%		p =	1%
n_1	pdf	cdf	pdf	cdf
0	0.0	0.0	8.1	8.1
1	0.0	0.0	20.5	28.6
2	0.0	0.0	25.7	54.3
3	0.1	0.1	21.5	75.8
4	0.3	0.5	13.4	89.2
5	0.9	1.3	6.7	95.9
6	1.8	3.1	2.7	98.6
8	5.4	11.9	0.3	99.9
10	9.6	29.1	0.0	100.0
12	11.6	51.8	0.0	100.0
14	10.0	72.9	0.0	100.0
16	6.4	87.5	0.0	100.0
18	3.1	95.3	0.0	100.0
20	1.2	98.5	0.0	100.0

Basel II: traffic lights

Zakres	n_1	Ξ	cdf(%)
	0	3	8.11
	1	3	28.58
	2	3	54.32
	3	3	75.81
	4	3	89.22
-	5	3+0.40	95.88
-	6	3+0.50	98.63
	7	3+0.65	99.60
	8	3+0.75	99.89
	9	3+0.85	99.97
	10+	4	99.99

Quantitative standards Basel II

e. The multiplication factor will be individual supervisory set bv authorities on the basis of their assessment of the quality of the bank's risk management system, subject to an absolute minimum of 3. Banks will be required to add to this factor a "plus" directly related to the ex-post performance of the model, thereby introducing a built in positive incentive to maintain the predictive quality of the model. The plus will range from 0 to 1 based on outcome of so-called the "backtesting."

Traffic lights: years 2019 - 2020

- § Historical simulation and parametric methods (normal and t-Student)
- § We sum the numbers of violations over 250 obs. windows from 28-03-2019 to 31-03-2020.
- § They are: 12, 13, 11 respectively.





Topic 7. Exercises

Exercise 7.1. Use the traffic light approach to the time series of a selected investment fund prices and 1% VaR computed with historical simulation. Compare your results with the results for the WIG index obtained in class.

Exercise 7.2. Propose a variant of the traffic light approach for the test window of 250 obs. and tolerance p=5%, analogous to Basel II for p=1%. Apply it to the selected investment fund and discuss the differences comparing to Ex. 1.

Exercise 7.3. A model for VaR with tolerance p is backtested on n = 250 obs. window. Let $\pi = n_1 / n$ be the VaR violation ratio, with n_1 being the number of violations. Determine 95% confidence interval (for left-tailed and two-tailed rejection regions) for n_1 and π , using functions dbinom/pbinom/qbinom from R. Assume:

- a. n = 250, p = 1%
- b. n = 250, p = 5%
- c. n = 100, p = 5%
- d. n = 100, p = 5%

Discuss the results.

Exercise 7.4* Use the traffic light approach to the series of a selected fund prices and 1% VaR computed from EWMA model. Compare the results with those from Ex. 7.1.
Coverage and independence tests

Formal model backtesting: What properties do we verify?

- 1. Proper coverage: Consistence of violation ratio with VaR tolerance
- 2. Independence of violations: Absence of violation clustering (actually noncorrelation of violations).

OLS regression tests

Coverage.

 $\eta_t = \pi + \varepsilon_t$ $H_0: \pi = p$

Independence.

$$\begin{split} \eta_t &= \pi + \rho \eta_{t-1} + \varepsilon_t \\ H_0: \rho &= \mathbf{0} \end{split}$$

Independent coverage.

 $\eta_t \sim \pi + \rho \eta_{t-1} + \varepsilon_t$ $H_0: \pi = p \land \rho = \mathbf{0}$

Warning. This approach is methodologically incorrect since ε_t is not normally distributed.

	HS	Norm.	t-stud.				
Coverage							
$\hat{\pi}$	0,012	0,012	0,004				
p-value	0,772	0,772	0,134				
Independence							
ρ	-0,012	-0,012	-0,004				
p-value	0,848	0,848	0,949				
In	depende	nt covera	ige				
$\hat{\pi}$	0,012	0,012	0,004				
ρ	-0,012	-0,012	-0,004				
p-value	0,940	0,940	0,330				

Backtesting of 1% VaR for WIG.

Binomial distribution test for coverage

- § Assuming that the model perfectly predicts VaR with tolerance p, the numer of violations should follow the binomial distribution B(n, p).
- § For $H_0: \pi = p$ (equiv. $n_1 = np$), we construct 95% confidence intervals, ie. intervals of H_0 -nonrejection. $[F_{B(n,p)}(0.025) \le n_1 \le F_{B(n,p)}(0.975)]$.

					Cdf's	s of	B (2	50 , p)) ar	nd H	o-no	nrej	ectic	n in	terv	als					
n_1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
p = 0.01	0.08	0.29	0.54	0.76	0.89	0.96	0.99	1.00	1.00	1.00	1.00	1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H_0																					
p = 0.02	0.01	0.04	0.12	0.26	0.44	0.62	0.76	0.87	0.93	0.97	0.99	1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
H_0																					
p = 0.05	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.06	0.12	0.19	0.29	0.4	0.52	0.63	0.73	0.81	0.88	0.92	0.95	0.97	0.99
H_0																					

- § Given fixed *n*, the widths of intervals grow with *p*.
- § For close *p*'s, the intervals overlap significantly.

Kupiec test: unconditional coverage

Kupiec test (of unconditional coverage) is the likelihood ratio test applied for the binomial distribution.

§ If $\eta_t \sim IID$ **Bernoulli**(α), then the likelihood of α conditional on n_1 and n_0 :

$$\mathcal{L}(\alpha|n_0, n_1) = \binom{n_0 + n_1}{n_1} \alpha^{n_1} (1 - \alpha)^{n_0}$$

§ Thus, the null hypothesis H_0 , that the probability of VaR violation is p:

$$H_0: P(\eta_t = 1) = p$$

§ can be verified using the *likelihood ratio* test: $\int (m|n, n) = m^{n_1}(1)$

$$LR_{UC} = \frac{\mathcal{L}(p|n_0, n_1)}{\mathcal{L}(\pi|n_0, n_1)} = \frac{p^{n_1}(1-p)^{n_0}}{\pi^{n_1}(1-\pi)^{n_0}}$$

§ Under H_0 : -2ln(LR_K) is asymptotically χ^2 (1) distributed.

Christoffersen Test (ver. 1): independence

Christoffersen Test 1.

Under the null hypothesis, we assume that the VaR violation at time t is independent of the violation at time t - 1:

$$H_0: P(\eta_t = 1 | \eta_{t-1} = 0) = P(\eta_t = 1 | \eta_{t-1} = 1) [= P(\eta_t = 1)]$$

§ Denote by n_{ij} the numer of observations where $\eta_{t-1} = i$ and $\eta_t = j$:

$$n_{ij} = #(\eta_{t-1} = i \text{ and } \eta_t = j)$$

§ Denote by π_0 and π_1 the ratios of violations after nonviolation and violation respectively:

$$\pi_0 = \frac{n_{01}}{n_{00} + n_{01}}$$
 and $\pi_1 = \frac{n_{11}}{n_{10} + n_{11}}$

Christoffersen Test (ver. 1): independence

Christoffersen Test 1.

Under the null hypothesis, we assume that the VaR violation at time t is independent of the violation at time t - 1:

$$H_0: P(\eta_t = 1 | \eta_{t-1} = 0) = P(\eta_t = 1 | \eta_{t-1} = 1) [= P(\eta_t = 1)]$$

§ Test statistics:

$$LR_{\rm Ch1} = \frac{\pi^{(n_{01}+n_{11})}(1-\pi)^{(n_{00}+n_{10})}}{\pi_0^{n_{01}}(1-\pi_0)^{n_{00}}\pi_1^{n_{11}}(1-\pi_1)^{n_{10}}} = \frac{\mathcal{L}(\pi|n_{00},n_{01}) \times \mathcal{L}(\pi|n_{10},n_{11})}{\mathcal{L}(\pi_0|n_{00},n_{01}) \times \mathcal{L}(\pi_1|n_{10},n_{11})}$$

§ Under H0: $-2\ln(LR_{Ch1})$ is asymptotically $\chi^2(1)$ distributed.

Christoffersen Test (ver. 2): conditional coverage

Christoffersen Test 2

Under the null hypothesis, we assume that the VaR violation at time t is independent of the violation at time $t - \mathbf{1}$ and occurs with probability equal to tolerance p.

$$H_0: P(\eta_t = 1 | \eta_{t-1} = 0) = P(\eta_t = 1 | \eta_{t-1} = 1) = p$$

Note: Ch2 = Kupiec + Ch1 (asymptotiacally)

§ Test statistics is the product of LR_K and LR_{Ch1}

$$LR_{\rm Ch1} = \frac{p^{(n_{01}+n_{11})}(1-p)^{(n_{00}+n_{10})}}{\pi_0^{n_{01}}(1-\pi_0)^{n_{00}}\pi_1^{n_{11}}(1-\pi_1)^{n_{10}}} = \frac{\mathcal{L}(p|n_{00},n_{01}) \times \mathcal{L}(p|n_{10},n_{11})}{\mathcal{L}(\pi_0|n_{00},n_{01}) \times \mathcal{L}(\pi_1|n_{10},n_{11})}$$

§ Under H0: $-2\ln(LR_{Ch2})$ is asymptotically $\chi^2(2)$ distributed.

Backtesting - illustration

Number of observation: n = 2500 § Expected numer of violations: § *np* = **125** Actual numer of violations: *n*₁ = **124** § § Kupiec Test, UC test: H_0 Christoffersen Test (2), CC test: H_1 §



What is wrong with this VaR model?

Summary

- § Backtesting depends on VaR tolerance p and sample size n
- § Allows to reject bad VaR models
- § Three tests: Kupiec, Chris. (1 & 2)
- § Results for 1% VaR for WIG index from 04-05-2016 to 28-04-2017 (250obs.):

Mathod	Test					
Method	Kupiec	Christ. 1	Christ. 2			
Hist. sim.	\checkmark	\checkmark	\checkmark			
norm.	\checkmark	\checkmark	\checkmark			
t-Stud.	\checkmark	\checkmark	\checkmark			
EWMA norm.	\checkmark	\checkmark	\checkmark			
EWMA t-Stud.	\checkmark	\checkmark	\checkmark			
GARCH norm.	\checkmark	\checkmark	\checkmark			
GARCH t-Stud.	\checkmark	\checkmark	\checkmark			





Topic 7. Exercises cont.

Exercise 7.5 Conduct OLS regression based tests for VaR derived from the parametric normal model for a selected investment fund. Assume T = 1250, n = 250 and p = 0.05.

Exercise 7.6 Use Kupiec and Christoffersen tests for VaR derived from the parametric normal model for a selected investment fund. Assume T = 1250, n = 250 oraz p = 0.05. Compare results with Ex. 7.5.

Exercise 7.7 As in Ex. 7.6, but for: a. T = 250, n = 1250, p = 0.05b. T = 1250, n = 250, p = 0.01What are the differences?

Exercise 7.8* Use Kupiec and Christoffersen tests for VaR derived from the EWMA and GARCH models for a selected investment fund. Assume T = 1250, n = 250 oraz p = 0.05. Compare results with Ex. 7.6.

Power of backtests

- § Discussed tests allow to reject bad models.
- § Unfortunately, they provide little information on the quality of accepted models.

Question: How to assess the probability of accepting a bad model and how to control it?

Answer: Control the power of the applied test.

		decision	
		H_0	H_1
eality	H ₀	prob. $1 - \alpha$	Type I error (prob. α)
	H ₁	Type II error (prob. β)	power (prob. 1 – β)

Power of backtests: traffic light

- § Assume, that an M_p method perfectly estimates VaR with tolerance p, ie. VaR violations are *IID* **Bernoulli(**p**)**.
- § We backtest M_p using the traffic light approach metodą on a 250 obs. window.

	р	n_1	0 – 4	5 – 9	10 –	Test power
H_0	0.01		0.892	0.108	0.000	
	0.02		0.439	0.531	0.030	3%
H ₁	0.03	P(n ₁)	0.128	0.651	0.221	22%
	0.05		0.005	0.190	0.805	81%

- § A method that systematically overestimates VaR (whose actual tolerance is p > 0.01) can end up in geen or yellow zone with quite high probability.
- § The traffic light approach has low power if the actual tolerance is close to 1%.

Power of tests: binomial distribution

- § We construct H_0 -nonrejection intervals for:
 - p = 0.01, 0.02, 0.05,
 - *n* = 250, 500, 1000, 1500.
- § We assume significance $\alpha = 0.05$, thus 95% confidence intervals.

p	n = 250	n = 500	n = 1000	n = 1500
H_0 -	-nonrejection i	intervals for th	e numbers of v	iolations n_1
0.01	[1, 5]	[2, 9]	[5, 16]	[9, 22]
0.02	[2, 9]	[5, 16]	[13, 28]	[21, 40]
0.05	[7, 19]	[17, 34]	[38, 63]	[60, 91]

§ To distinguish methods with actual tolerances **0.01** and **0.02**, the test sample size must be significantly larger than 250 obs.

Analysis of Kupiec test power

- § Asymptotic distribution of $-2\ln(LR_K)$ is $\chi^2(1)$.
- § Thus we can derive H_0 -nonrejection intervals for various tolerances and test sample sizes (assuming significance $\alpha = 0.05$).

p (%)	n = 250	n = 250 n = 500		n = 1500
	H ₀ -nonreje	ction intervals for the	numer of violations n_1	
1	[1,6]	[2, 9]	[5, 16]	[9, 23]
2	[2, 9]	[5, 16]	[12, 29]	[21, 41]
5	[7, 19]	[17, 35]	[38, 64]	[60, 92]
		Power of the test for	$H_0: P(r \le VaR) = 0.01$	
2	0.243	0.544	0.782	0.888
5	0.969	1.000	1.000	1.000

- § Intervals are relatively wide for n = 250 and their widths decrease with n.
- § For small *n*, intervals overlap substantially.

ES backtesting: McNeil and Frey test; Berkowitz test

McNeil and Frey test for ES

Let's denote:

- τ : times of VaR violations, ie. τ : $r_{\tau} < VaR_{\tau}$,
- n_1 : numer of violations
- u_{τ} : distance from ES_{τ} , ie. $u_{\tau} = ES_{\tau} r_{\tau}$
- \bar{u} : average distance of u_{τ}
- S_u : u_{τ} standard deviation
- § The null hypothesis of MF test is:

$$H_0: E(u_\tau) = \mathbf{0}$$

§ Under H_0 , the test statistics:

$$MF = \frac{\overline{u}}{S_u / \sqrt{n_1}}$$

§ *MF* is asymptotically N(0,1) distributed. For small n_1 its distribution is approximated using *bootstrap*.

Historical simulation

- § Backtesting of 1% VaR and ES on 1250 obs.
- § Estimation windows 300 obs.
- § 16 violations (expected 12.5).
- § Kupiec test p-value: 0.34
- § Christ. (2) test p-value: 0.25
- § MF test p-value: 0.42





Summary

	Test							
Method	Binom. dist.	Kupiec	Christ. (2)	MF				
Hist. sim.	\checkmark	\checkmark	\checkmark	\checkmark				
norm.	X	×	×	X				
t-Stud.	\checkmark	\checkmark	X	\checkmark				
EWMA norm.	X	×	×	X				
EWMA t-Stud.	\checkmark	\checkmark	\checkmark	\checkmark				
GARCH norm.	X	×	X	X				
GARCH t-Stud.	\checkmark	\checkmark	\checkmark	\checkmark				

Bonus: testing entire predicted return distribution Berkowitz test (2001)

§ Presented models/methods provide predictions of entire return distributions:

pdf: $f_{T+1|T}(r)$ and cdf: $F_{T+1|T}(r)$

§ Given realized returns r_{T+1} , we can compute the probability integral transform (PIT, Rosenblatt '52):

$$PIT_{T+1|T} = \int_{-\infty}^{T+1} f_{T+1|T}(r) dr = F_{T+1|T}(r_{T+1})$$

§ If our model/method is well callibrated, i.e. accurately predicts VaR_p for all tolerance levels p, then:

$$PIT_{T+1|T} \sim U(0,1)$$

Bonus: testing entire predicted return distribution Berkowitz test (2001)

lf:

$$PIT_{T+1|T} \sim U(0,1)$$

then:

$$z_{T+1|T} = \Phi^{-1}(PIT_{T+1|T}) \sim IID N(0,1),$$

Berkowitz test. For:

 $z_{t+1|t} = \alpha + \rho z_{t|t-1} + \varepsilon_{t+1}$

we test:

 $H_0: \alpha = \mathbf{0} \land \rho = \mathbf{0}$

Topic 7. Exercises

Exercise 7.9. Perform McNeil and Frey test for the time series of a selected fund prices and ES computed using historical simulation, parametric methods (for normal and t-Student distributions), EWMA and GARCH on a test window of 250 obs. Assume tolerances p = 1% and p = 5%. Compare your results with those for WIG index.

Exercise 7.10. As in Ex. 7.9 but for estimation windows of 250 obs. and the test wondow of 1000 obs. What are the differences?

Exercise 7.11. Perform Berkowitz test for settings from Ex. 7.9.

Topics 5-7 presentation

Contents of the presentation:

a. <0p> Remain main informations about the fund.

b. <1.5p> Present 5% VaR for horizons from 1 to 4 days using:

- square root of time method (normal distribution)
- Cornish-Fisher expansion
- MC simulations from GARCH model

Present the results in a Table and on the graph..

- c. <2p> Compute the risk in 1-year horizon by comparing VaR and S-VaR (for stressed values of variance computed as 99 percentile from 21-day window) for 5% tolerance level and assuming normal distribution. Present the histogram of variance with the stressed and unconditional variance values.
- d. <2.5p> Compute % change in your fund portfolio value in the scenario of the initial months of COVID-19 pandemics:
 - commodity price decline by 50%, but precious metals price increase by 25%
 - stock price declines by 10% in developed countries and 15% in emerging economies (Note: PL is classified as EME)
 - depreciation of EME currencies by 10%
 - yield curve downward shift by 100 bp.

Present the structure of your portfolio, sensitivity analysis and the calculated impact on portfolio value. Compare the results with realized change in the fund value in the period: 1.03-31.05.2020.

- e. <3p> Backtest 1% VaR with evaluation window of 250 observations (traffic lights, Kupiec test, Christoffersen, McNeil-Frey) for:
 - Historical simulation / normal distribution
 - Cornish-Fisher
 - EWMA or GARCH

Present p-values of tests as well as VaR exceedance plots. Which model is the best?

Additionally, 1 p. for the quality of presentation and the speech. Time limit: 5 minutw. Avoid a large number of slides (7 slides is a good choice). Presentation in pdf file entitled *SurnameName.pdf* download to MT.