Optimization. A first course on mathematics for economists
Problem set 8: Difference equations

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8.1 Let the demand of a certain commodity be given by $D(p_t) = \alpha - \beta p_t$ and its supply by $S(p_t) = \gamma + \delta p_t$, where $\alpha, \beta, \delta > 0$. Assume the price p adjustes from one period to the next according to the stocks cummulated by the sellers, in the following way

$$p_{t+1} = p_t - r(S(p_t) - D(p_t)) \tag{1}$$

(a) determine the trajectory of the price along time

(b) study the properties of the trajectory when $r = 0.1, \beta = 1, \delta = 15$

(c) study the properties of the trajectory when $r = 0.3, \beta = 2, \delta = 6$

8.2 Solve the following equation

$$x_t = \frac{1}{2}x_{t-1} + 3 \tag{2}$$

for $x_0 = 2$.

8.3 Solve the following equation

$$y_t = -3y_{t-1} + 4 \tag{3}$$

for $y_0 = 2$.

8.4 Solve the following equation

$$x_t = -\frac{1}{2}x_{t-1} + 3 \tag{4}$$

for $x_0 = 2$.

8.5 Solve the following equation

$$y_t = 3y_{t-1} + 4 \tag{5}$$

for $y_0 = 2$.

8.6 Consider an individual contracting a mortgage for B€ at $t = 0$. Suppose that (i) the interest rate r is constant along time, (ii) repayment per period z is also constant until mortgage is paid off after T periods. The principal b_t on the loan in period t is given by

$$b_t = (1+r)b_{t-1} + z, \text{ with } b_0 = B, b_T = 0 \tag{6}$$

(a) Solve equation (6)

(b) Compute B and give an economic meaning to the expression obtained

(c) Compute z and give an economic meaning to the expression obtained

(d) Compute the principal repayment in period t, b_t and give an economic meaning to the expression obtained

8.7 Solve the following difference equations and study the solution paths.

(a) $x_{t+2} + 3x_{t+1} - \frac{7}{4}x_t = 9$, with $x_0 = 0, x_1 = 6$

(b) $x_{t+2} - x_{t+1} + \frac{1}{4}x_t = 2$, with $x_0 = 0, x_1 = 6$

(c) $x_{t+2} + 2_x t + 1 + x_t = 9(2)^t$, with $x_0 = 0, x_1 = 6$

8.8 Consider an economy whose GDP at time t, Y_t is defined as

$$Y_t = C_t + I_t \tag{7}$$

where I_t denotes investment, and C_t denotes consumption. Suppose that

- consumption is determined as

$$C_{t+1} = aY_t + b \tag{8}$$

where $a, b > 0$.

- Investment is defined as proportional to the change in consumption

$$I_{t+1} = c(C_{t+1} - C_t) \tag{9}$$

with $c > 0$.

Derive the expression of the GDP as a second-order difference equation and assess the stability of the solution.