

Optimization. A first course on mathematics for economists

Problem set 4: Classical programming

Xavier Martinez-Giralt

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4.1 Solve the following problem:

$$\min_{x_1, x_2} 2x_1^2 + x_2^2 \text{ s.t.}$$
$$x_1 + x_2 = 1$$

Give a geometric interpretation to the solution.

4.2 Suppose we have a distribution center that distributes goods to several retail outlets in a city. There are two routes to go from the distribution center to the city A and B . The cost of shipping x units using route A is ax^2 , $a > 0$. The cost of shipping y units using route B is by^2 , $b > 0$.

(a) Suppose Q units have to be distributed. Determine how they must be allocated to routes A and B to minimize the total shipping cost.

(b) How does the cost change if Q increases by $r\%$?

4.3 An individual has some savings that wants to invest. He wants to minimize risk and obtain an expected return of 12%. There are three mutual funds available yielding expected returns of 10%, 10%, and 15% respectively. Let x , y , and z be the proportion of the savings invested in each of the three funds. The financial experts report that the measure of risk is given by

$$400x^2 + 800y^2 + 200xy + 1600z^2 + 400yz$$

Determine how the individual should distribute his savings among the three funds minimizing the risk.

4.4 An individual has preferences defined over three consumption goods x , y , z . This preferences are represented by means of an utility function

$$U(x, y, z) = 5 \ln x + 8 \ln y + 12 \ln z$$

Unit prices of the goods are $p_1 = 10\text{€}$, $p_2 = 15\text{€}$, $p_3 = 30\text{€}$. The income of the individual is $m = 3000\text{€}$.

Find the consumption bundle maximizing the utility of the individual.

- 4.5 A firm uses three inputs, u, v, w , to produce a certain good. Its production function is

$$Q(u, v, w) = 36u^{1/2}v^{1/3}w^{1/4}$$

The unit prices of the inputs are $p_u = 25\text{€}$, $p_v = 20\text{€}$, $p_w = 10\text{€}$.

- (a) Find the levels of the inputs maximizing the output, given that the firm faces a budget constraint of $m = 78000\text{€}$
- (b) Use the envelope theorem to assess how much can the firm increase the production if its budget increases to 80000€ .

- 4.6 Let $f(x_1, x_2) = x_1x_2$. Solve the following problem:

$$\begin{aligned} \min_{x_1, x_2} x_1 + x_2 \text{ s.t.} \\ x_1 + 4x_2 = 16 \end{aligned}$$

- 4.7 Let $f(x_1, x_2, x_3) = x_1x_2x_3$, $h_1(x, y, z) \equiv x_1^2 + x_2^2 = 1$, $h_2(x, y, z) \equiv x_1 + x_3 = 1$. Characterize the set of candidate solutions of the following problem:

$$\begin{aligned} \min_{x_1, x_2, x_3} x_1 + x_2x_3 \text{ s.t.} \\ x_1^2 + x_2^2 = 1 \\ x_1 + x_3 = 1 \end{aligned}$$