

**Will the “True” Labor Share Stand Up?
An Applied Survey on Labor Share Measures**

Online Appendices

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A The Importance of the Labor Share in Economic Applications

The purpose of this appendix is to discuss the potential consequences of using these diverse series interchangeably in empirical applications. We consider three applications: (1) growth accounting, (2) technology-labor share VAR analysis, and (3) estimation of New Keynesian Phillips curves (repeated from the main text). These three were chosen not only to reflect their broad popularity in various literatures, but also because they help reveal and substantiate some of the differences in the series discussed earlier.

A.1 Application I: Growth Accounting

We begin with growth accounting, which is a widely-used exercise in macroeconomics, development and business-cycle analysis.¹ This exercise decomposes economic growth into that due to factor accumulation, and technical progress (which is derived residually). The standard growth accounting equation can be written as:²

$$\Delta \text{tfp}_t = \Delta y_t - \tilde{\alpha}_t \Delta k_t - (1 - \tilde{\alpha}_t) \Delta l_t, \quad (\text{A.1})$$

where all variables are in logs, and where $\tilde{\alpha}_t$ and tfp_t denote the (potentially time-varying) capital share and log total factor productivity at time t , respectively.

We already know that labor shares are time-varying and have different properties across variants. Accordingly, this should be reflected in how we implement growth accounting. With this in mind, the extraction of TFP can then be done in the following ways:

1. Common Input Factors and Outputs

Derive TFP across different labor share measures based on *common* input factors and *common* inputs:

Y_t^1 : GDP in constant USD [*NIPA Table 1.6*];

K_t^1 : Chain-Type Quantity Index for the Net Stock of Fixed Assets [*FAT Table 1.2*];

¹For a discussion see Fernald (2015).

²This decomposition assumes a constant-return Cobb Douglas production under competitive factor markets.

L_t^1 : Full-Time Equivalent Employees plus Self-Employed in all domestic industries [NIPA Tables 6.5 and 6.7].

2. Common Input Factors But Definitionally Consistent Outputs

Derive TFP across different labor share measures based on common input factors but with output measures related to the specific labor-share measure:

Y_t^1 : GDP in constant USD [NIPA Table 1.6]

Y_t^2 : Real Gross Value Added in the private sector [NIPA Table 1.3.6];

Y_t^3 : Real Gross Value Added in the non-farm business sector [NIPA Table 1.3.6];

K_t^1 and L_t^1 as above.

3. Definitionally Consistent Factors and Outputs

Derive TFP across different labor share measures based on input factors and output measures related to the specific labor-share measure:

$Y_t^1 - Y_t^3, K_t^1, L_t^1$: as above;

K_t^2 : Chain-Type Quantity Index for the Net Stock of Fixed Assets in the private sector [FAT Table 1.2]

K_t^3 : Chain-Type Quantity Index for the Net Stock of Fixed Assets in the non-farm business sector [also FAT Table 1.2];

L_t^2 : Full-Time Equivalent Employees plus Self-Employed in the private sector [NIPA Tables 6.5 and 6.7].

L_t^3 : Full-Time Equivalent Employees plus Self-Employed in the non-farm business sector [NIPA Tables 6.5 and 6.7].

Observe that for **NaiveGDP**, **PI-GDP**, **PI₂-GDP** and **SE-GDP**, all these approaches boil down to the same growth accounting scenario because our reference measures of GDP, capital, and labor are then also definitionally consistent.

Within each of these three cases, we assume that factor shares are time-varying following a Törnquist index:

$$\tilde{\alpha}_{j,t} = \frac{\alpha_{j,t} + \alpha_{j,t-1}}{2},$$

where j denotes the particular labor share variant used (e.g., **NaiveGDP**, **PI-GDP**, etc.).

Our purpose therefore is to examine the scope for mis-measurement of TFP (growth and levels) when factor income shares vary and when differences in shares are compounded with those of output and the factors. **Figure A.1** shows the cumulated TFP levels for time-varying income shares, where the factors are assumed constant across labor share definitions (the first row), where the output definitions are additionally allowed to change (second row), and where both the inputs and outputs are allowed to change consistent with the underlying labor share definition (final row).

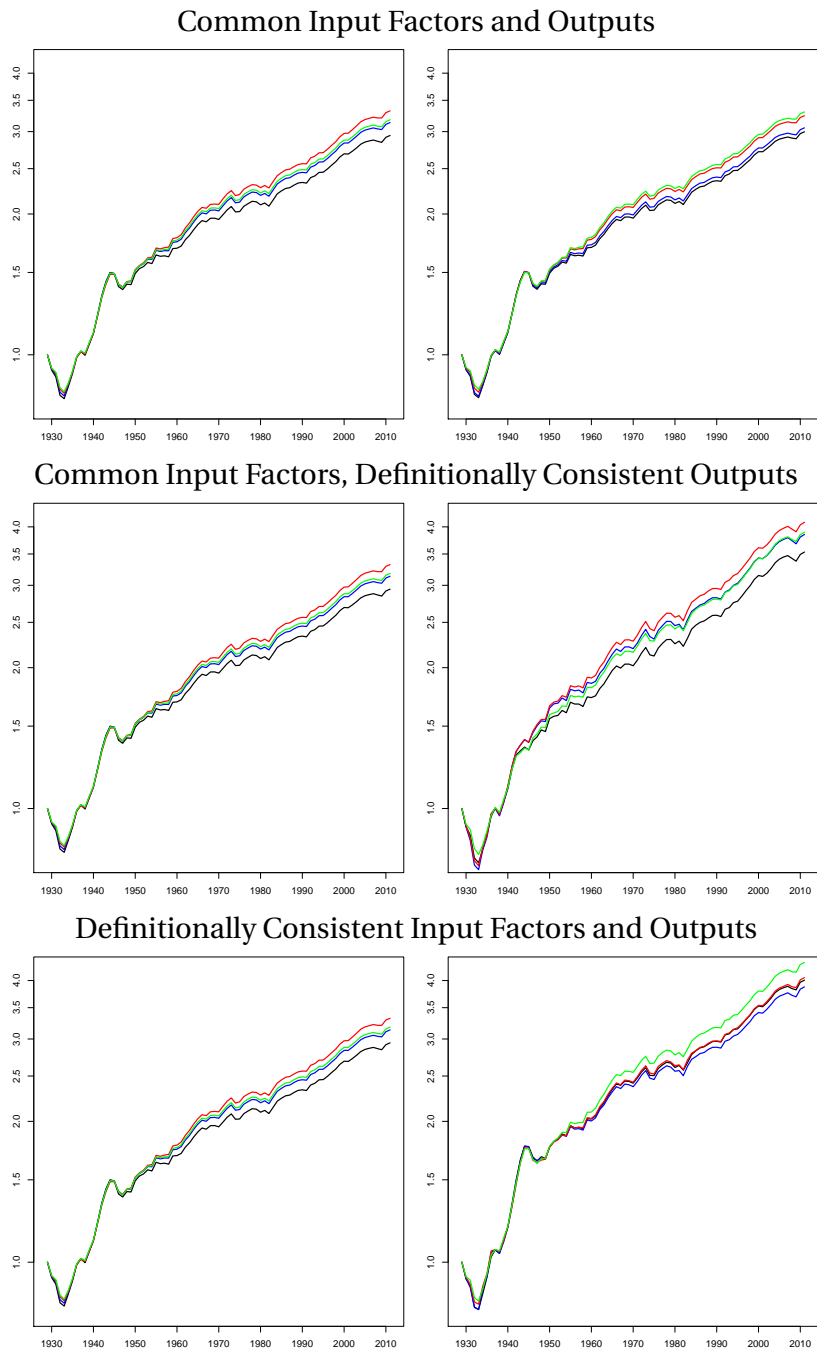
In terms of shape, all series broadly conform to what is commonly understood to be the story behind US TFP (e.g., Fernald (2007), Shackleton (2013)): exceptionally high TFP growth in the mid 1930-1940s, the consolidation of those gains in the decades after WW II, followed by a period of slower residual productivity growth (often dated to the early 1970s), and the acceleration in productivity towards the end of the sample.

There are, though, certain level differences between the TFPs generated with the use of the alternate factor share series. Some specific discrepancies in dynamics are worth noting too. For example, productivity and TFP growth are often considered to have exhibited a broken trend in the early 1970s (e.g., Fernald, 2007). Whilst this is clearly visible for most of the series, it is less apparent for the GVA series.³

Table A.1 shows the cumulative change of TFP based on time-varying factor income shares. There are indeed substantial cumulative discrepancies. To illustrate, whilst **NaiveGDP** grew by 108% over the whole sample, **SE-GVA-NF** grew by 140% (in the last accounting scenario).

³The absence of a slight hump in the TFP level in the mid 1940s (see middle panel, rhs graph) is caused by the fact that real GDP/GVA grew at slightly different rates. For example, the most spectacular difference was in 1946 when GDP fell by 10% while GVA by less than 1%. Note that this does not reappear in the bottom row rhs graph since then we adjust the inputs consistently with the labor share definition. Note also that it is the period after WWII so there was a substantial shift between sectors (government vs private).

Figure A.1: Cumulative TFP based on Time-Varying Factor Shares (1929=1)



Notes:

Left panel: Naive-GDP, **PI-GDP**, **PI₂-GDP**, **SE-GDP** ; Right panel: Naive-GVA, **Naive-GVA-NF**, **SE-GVA-NF**, **SE-GVA**

A log-scale for the level of TFP is used in these graphs for legibility.

Table A.1: Cumulative Change of TFP, Based on Time-Varying Factor Shares (In %)

	Naive-GDP	PI-GDP	PI ₂ -GDP	SE-GDP	Naive-GVA	Naive-GVA-NF	SE-GVA-NF	SE-GVA
	Common Input Factors and Outputs							
1929-2011	108	114	120	116	110	112	118	120
1929-1945	40	40	40	40	40	40	41	40
1945-1970	27	31	34	32	27	28	32	34
1970-2011	41	43	46	44	43	43	45	46
	Common Input Factors, Definitionally Consistent Outputs							
1929-2011	108	114	120	116	126	135	141	136
1929-1945	40	40	40	40	29	33	33	29
1945-1970	27	31	34	32	41	46	50	48
1970-2011	41	43	46	44	56	56	59	59
	Definitionally Consistent Input Factors and Outputs							
1929-2011	108	114	120	116	139	136	140	148
1929-1945	40	40	40	40	57	57	56	56
1945-1970	27	31	34	32	31	30	32	38
1970-2011	41	43	46	44	51	49	51	54

Interestingly, we also find that the TFP deviations for all labor share specifications against **NaiveGDP** have been gradually increasing since World War II, see **Figure A.1** and **Table A.2**.⁴ This is driven by the fact that the post-war period was characterized by rapid physical capital accumulation, and hence the underestimation of the labor share (equivalently, overestimation of the capital share) in the **Naive-GDP** case has systematically led to an overstating of capital's contribution to GDP growth, at the cost of understating the role of TFP. By this logic, it should not be surprising that the relatively highest labor share **PI₂-GDP** implies also the relatively

⁴The observed divergence between the GDP-based and GVA-based series in the initial period 1929–1945 in the last two accounting scenarios is due to real GDP/GVA growing at different rates in the 1940s.

Table A.2: Cumulative Deviation from TFP Based on NaiveGDP (In %)

	Naive-GDP	PI-GDP	PI ₂ -GDP	SE-GDP	Naive-GVA	Naive-GVA-NF	SE-GVA-NF	SE-GVA
	Common Input Factors and Outputs							
1929-2011	0	6	12	8	2	4	10	12
1929-1945	0	0	0	0	0	0	1	0
1945-1970	0	4	8	5	0	2	5	7
1970-2011	0	2	5	2	1	2	4	4
	Common Input Factors, Definitionally Consistent Outputs							
1929-2011	0	6	12	8	18	27	33	28
1929-1945	0	0	0	0	-11	-7	-7	-11
1945-1970	0	4	8	5	15	19	23	21
1970-2011	0	2	5	2	15	15	17	18
	Definitionally Consistent Input Factors and Outputs							
1929-2011	0	6	12	8	31	28	32	40
1929-1945	0	0	0	0	17	17	16	16
1945-1970	0	4	8	5	5	3	6	11
1970-2011	0	2	5	2	9	8	10	13

strongest TFP growth.

A.2 Application II: Technology Shocks and the Labor Share

Let us now pass to the short-run question of examining the impact of exogenous technology shocks on the labor share. The motivation for undertaking such an exercise is the following. First, it is worthwhile to verify if the apparently consistent short-run properties of all considered labor share measures carry forward to applied econometric studies of the business cycle. The question of the impact of technology shocks seems a reasonable first step in this direction. Second, as argued above the labor share switches from being countercyclical in the short run to being procyclical in the medium run. Rios-Rull and Santaella-Llopis (2010) found an overshooting response of the labor share to technology shocks, consistently with its short-run countercyclicality and a positive correlation of output with lagged labor shares. We shall verify if this property holds for various labor share definitions.

The current analysis is based on quarterly data spanning 1948q1-2013q1. Our technological shock variable is TFP growth, taken from Fernald (2012). Fernald's TFP measures are superior to the ones derived in the previous section because they distinguish between heterogeneous physical capital and labor types, whose unit productivities are inferred from data on relative prices.⁵ Another advantage of using Fernald's TFP time series is that they also include TFP adjusted for capacity utilization, which constitutes an important wedge between the available inputs and currently produced output. Capacity utilization (varying machine hours, labor hoarding, etc.) can indeed partially absorb technological shocks before they are transmitted to changes in factor shares.

Let us first, though, elaborate on how we view an "aggregate" TFP shock. Consider the general CES production function:

$$Y = \left[\alpha (\Gamma_K K)^{\frac{\zeta-1}{\zeta}} + (1 - \alpha) (\Gamma_L L)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}} \quad (\text{A.2})$$

where Y represents real output, K is the capital stock, L is the labor input and $\zeta \geq 0$

⁵In his TFP computations, Fernald (2012) has interpolated annual data on factor shares from the BLS multifactor productivity database, claiming that his "results were little affected in experiments with other reasonable choices, such as using national accounting data". At this, it is reassuring that our findings corroborate the robustness of TFP calculations to changes in factor share definitions, especially in the short run. See also the work of Basu et al. (2006).

is the elasticity of factor substitution.

Terms Γ_j capture the level of technical progress with $d \log \Gamma_j = \gamma_j$ its rate of growth. Which type(s) of technical progress are present in the economy (and whether $\zeta \geq 1$) matters for how the factor income shares evolve. We can re-express (A.2) in per-capita log form and apply a Taylor-series expansion around the point $\zeta = 1$ (following Kmenta (1967); Klump et al. (2012)):

$$y = \alpha k + \Lambda k^2 + \underbrace{\alpha \left[1 + \frac{2\Lambda}{\alpha} k \right] \gamma_K + (1 - \alpha) \left[1 - \frac{2\Lambda}{(1 - \alpha)} k \right] \gamma_N + \Lambda [\gamma_K - \gamma_N]^2}_{\Phi = \log(TFP)} \quad (\text{A.3})$$

where $y = \log[\frac{Y}{L}]$, $k = \log[\frac{K}{L}]$, $\Lambda = \frac{(\zeta - 1)\alpha(1 - \alpha)}{2\zeta}$.

If $\zeta \rightarrow 1$ then $y = \alpha \gamma_K + (1 - \alpha) \gamma_N + \alpha k$ (i.e., the under-identified Cobb-Douglas form). Otherwise the log of TFP is an average of capital and labor augmenting technologies (with the weights determined by the capital-labor ratio and the income shares). In the absence of careful estimation of the production relationships (inter alia, Klump et al. (2007)), we do not observe Φ in the factor-augmenting case (for an econometric discussion see León-Ledesma et al. (2010)). Nonetheless, the structure of (A.3) illuminates how we might think of general TFP shocks – namely as driven mostly by labor-augmenting components given the typical value of $\alpha < 0.5$ and $\sigma < 1$ (see Chirinko (2008); Klump et al. (2007)). For instance $\frac{\partial \Phi / \partial \gamma_L}{\partial \Phi / \partial \gamma_K} > 1$ if $\alpha + 2(1 + \gamma_K - \gamma_L)\Lambda < \frac{1}{2}$. For a wide range of values this will typically hold, example for $\alpha = 0.33$, $\zeta = 0.6$ and $\gamma_K \approx \gamma_L$.

Given this, the relative capital-to-labor income share, given competitive factor markets and profit maximization, can be expressed as

$$\Theta = \frac{w L}{r K} = \frac{1 - \alpha}{\alpha} \left(\frac{\Gamma_K K}{\Gamma_L L} \right)^{\frac{1 - \zeta}{\zeta}}, \quad (\text{A.4})$$

where r_t and w_t denote the user cost (or marginal productivity) of capital and the real wage, respectively.

Otherwise, factor income shares are changed by movements in capital per worker or biases in technical change or relative movements in factor utilization. The direction of the effect, however, depends on the value of the substitution elasticity, and, in the case of technology shocks, on their source (i.e., whether they augment capital

or labor):

$$\begin{aligned} &< 0 \text{ for } \zeta < 1 \\ \frac{\partial \Theta}{\partial (\Gamma_L/\Gamma_K)}, \frac{\partial \Theta}{\partial (K/L)} &= 0 \text{ for } \zeta = 1 \\ &> 0 \text{ for } \zeta > 1 \end{aligned} \tag{A.5}$$

Accordingly, from (A.5), if the elasticity of substitution is $\zeta < 1$, a shock which is net labor-augmenting will reduce the labor share.

A.2.1 ARDL Model

To assess the impact of exogenous technology shocks on the labor share, we first estimated a range of simple autoregressive distributed lag models:

$$x_t = \mu + \rho x_{t-1} + \sum_{i=0}^k \beta_i \Delta \text{tfp}_{t-i} + \varepsilon_t \tag{A.6}$$

where $x_t = \log(LS_t)$ and Δtfp_{t-k} is TFP growth (difference in log TFP levels) lagged k quarters. **Table A.3** shows the results.

First, we find a negative contemporaneous correlation between the labor share and technological shocks. The correlation with lagged TFP growth is positive, though. This is suggestive of a non-monotonic, overshooting dynamics of the labor share following a TFP shock.⁶

Second, we find that the effect of the technological shock is highest for the **BLS** and **SE-GDP** series, and lowest for **PI₂-GDP**, regardless of whether TFP shocks are capacity-adjusted or not.

Third, we extended model (A.6) to allow for the presence of *asymmetric* effects of technological shocks on the labor share. Thus we have checked whether the labor share reacts differently to positive and negative TFP shocks. Such results were

⁶Using rolling window estimation of equation (A.6) and its counterpart for HP-filtered series we observe a slight decrease in the contemporaneous correlation between the labor share and technological shocks over time.

obtained by splitting Δtfp_{t-1} into

$$\Delta\text{tfp}_{t-1}^+ = \Delta\text{tfp}_{t-1}\mathcal{I}(\Delta\text{tfp}_{t-1} > 0)$$

$$\Delta\text{tfp}_{t-1}^- = \Delta\text{tfp}_{t-1}\mathcal{I}(\Delta\text{tfp}_{t-1} < 0)$$

where \mathcal{I} is the indicator function. Under this specification, it is still estimated that $\beta_0 < 0$, i.e., the immediate effect of technology shocks is still to diminish the labor share. On the other hand, we also find that lagged (non-capacity-adjusted) technological shocks are positively correlated with the labor share if they are negative, and essentially uncorrelated if they are positive. This means that negative technological shocks tend to increase the labor share only temporarily (majority of the immediate negative effect disappears after one period), whereas positive technological shocks depress the labor share permanently (or at least for a longer time), and the overshooting dynamic is absent.

This result is largely driven by capacity adjustment, however. We do not find any evidence for asymmetric correlation between TFP shocks adjusted by capacity utilization and the labor share. Hence, it can be concluded that negative TFP shocks may appear as temporary only because they induce substantial declines in capacity utilization. If these declines are properly accounted for, negative TFP shocks tend to increase the labor share permanently as well. The numbers are fairly symmetric across all labor share specifications, albeit again, they are somewhat larger for the **BLS** and **SE-GDP** series, and smaller for **PI₂-GDP**.

A.2.2 VAR Analysis

A more sophisticated approach to assessing the impact of exogenous technology shocks on the labor share requires the researcher to allow for mutual impact of both variables. In its simplest form, such relationships can be analyzed by the means of a bivariate VAR model:

$$z_t = a_0 + \sum_{i=1}^p \Phi_i z_{t-i} + u_t \tag{A.7}$$

where $z_t = \begin{bmatrix} \Delta\text{tfp}_t \\ x_t \end{bmatrix}$ is the vector of jointly determined dependent variables and u_t is a 2×1 vector of disturbances. Lag length p shall be selected according to the

Table A.3: ARDL Model with TFP

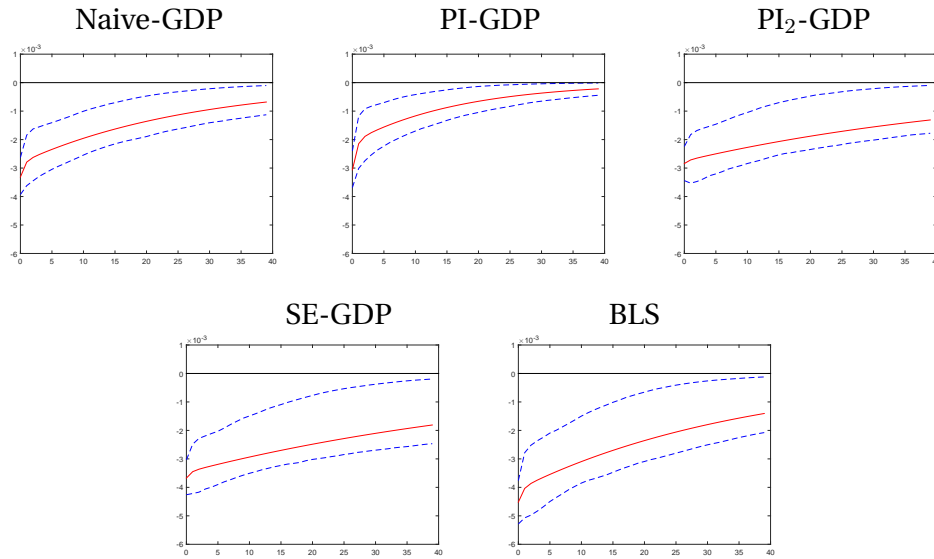
	Naive-GDP	PI-GDP	PI ₂ -GDP	SE-GDP	BLS
NO CAPACITY ADJUSTMENT					
$\hat{\rho}$	0.955***	0.943***	0.987***	0.995***	0.981***
$\hat{\beta}_0$	-0.388***	-0.357***	-0.331***	-0.431***	-0.528***
$\hat{\beta}_1$	0.109**	0.152***	0.0704*	0.098**	0.141**
CAPACITY ADJUSTMENT					
$\hat{\rho}$	0.966***	0.949***	0.987***	0.991***	0.982***
$\hat{\beta}_0$	-0.252***	-0.233***	-0.196***	-0.289***	-0.351***
$\hat{\beta}_1$	0.155***	0.188***	0.140***	0.172***	0.215***
NO CAPACITY ADJUSTMENT + ASYMMETRIC LAG STRUCTURE					
$\hat{\rho}$	0.953***	0.947***	0.993***	1.00***	0.985***
$\hat{\beta}_0$	-0.391***	-0.359***	-0.335***	-0.435***	-0.530***
$\hat{\beta}_1^-$	0.324***	0.307***	0.284***	0.267**	0.265*
$\hat{\beta}_1^+$	0.006	0.0809	-0.028	0.019	0.084
CAPACITY ADJUSTMENT + ASYMMETRIC LAG STRUCTURE					
$\hat{\rho}$	0.960***	0.956***	0.989***	0.995***	0.985***
$\hat{\beta}_0$	-0.240***	-0.212***	-0.190***	-0.285***	-0.342***
$\hat{\beta}_{1,-}$	-0.079	-0.066	-0.094	-0.116	-0.182
$\hat{\beta}_{1,+}$	0.035	0.105*	0.007	0.007	0.056

Note: subscripts ***, ** and * denote the rejection of null about parameter's insignificance at 1%, 5% and 10% significance level, respectively. The constant estimated is suppressed for brevity.

BIC criterion.

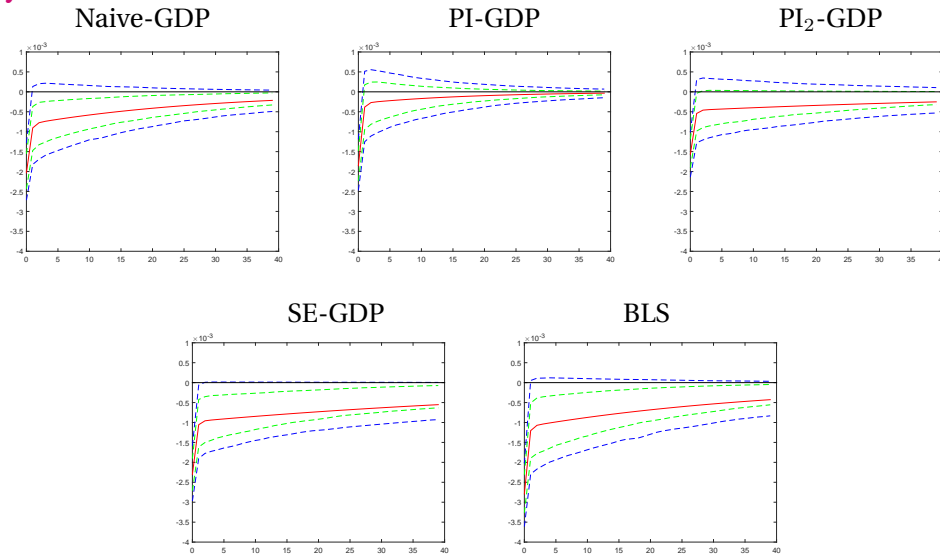
To analyze the dynamic response of the labor share to a technological shock we use orthogonal impulse response functions. Since residuals from equations in VAR models can be correlated, the standard IRF analysis does not include such information and, as a result, cannot generate the true trajectories. Therefore, covariance between residuals is taken into consideration via a Cholesky decomposition. Our ordering of variables corresponds to the ARDL approach and, correspondingly, Δtfp_t is set as the first variable in the system.

Figure A.2: Response of the Labor Share to a Technology Shock, no Capacity Adjustment



Notes: Bootstrapped 95% confidence bands shown in blue dotted lines.

Figure A.3: Response of the Labor Share to a Technology Shock Adjusted for Capacity Utilization



Notes: Bootstrapped 95% (68%) confidence bands shown in blue (green) dotted lines.

We find that in both sets of IRFs (**Figure A.2–Figure A.3**), the effect of a temporary TFP shock is to reduce the labor share. These results are in line with our earlier theoretical reasoning which points out that TFP shocks are typically relatively more labor- than capital-augmenting, and that capital and labor are gross complements, $\zeta < 1$. As with the ARDL case, however, we might speculate that some fraction of any technological improvement partly complements the existing capital stock or labor input, and partly raises utilization rates. This latter possibility disguise some of the identification of the technological shock's effect on factor shares and explains why the response to utilization-adjusted TFP shocks is generally much smaller and, at the 95% confidence level, only significant in the first period.

There are also marked differences in the speed of reversion, with **GDPPI** and (largely speaking) **NaiveGDP** having returned to their base within a 10 year horizon. For the other series, the effect is highly protracted and stretches into the domain of medium-term business cycles. Clearly in general equilibrium models where the labor share plays a non-trivial role (as for example in labor bargaining models) this differential speed of reversion of income shares from technology shocks will be very important.

A.3 Application III: New Keynesian Phillips Curves

Our final application is in the field of inflation modelling. As in Galí and Gertler (1999) and subsequent literature, the New-Keynesian Phillips Curve literature assumes staggered price setting under imperfect competition, where a fraction θ of firms do not change their prices in any given period. The remaining firms set prices optimally as a fixed mark-up, μ , on discounted expected marginal costs. When re-setting, firms also take into account that the price may be fixed for many future periods, yielding the optimal reset price p_t^* (see Tsoukis et al. (2011) for a comprehensive survey)

$$p_t^* = (1 - \theta\beta) \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k [mc_{t+k}^n + \mu] \quad (\text{A.8})$$

where mc^n is (the log of) nominal marginal costs, β is a discount factor, and \mathbb{E}_t is the expectation operator. The overall price level is then a weighted average of lagged and reset prices, $p_t = \theta p_{t-1} + (1 - \theta) p_t^*$. Given $mc_t^r \equiv mc_t^n - p_t$, and constant marginal

costs across firms, the familiar “New Keynesian Phillips Curve” (NKPC) emerges,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda (mc_t^r + \mu) \quad (\text{A.9})$$

where $\pi_t = p_t - p_{t-1}$ is inflation and $\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta}$ represents the reduced-form “slope”.

Additionally, it is often assumed that of the $1 - \theta$ price-re-setting firms a fraction, ω , set their price according to lagged inflation. This implies a NKPC with an intrinsic expectations component:

$$\pi_t = \gamma_f \mathbb{E}_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda (mc_t^r + \mu) \quad (\text{A.10})$$

where $\phi = \theta + \omega [1 - \theta (1 - \beta)]$, $\gamma_f = \frac{\theta\beta}{\phi}$, $\gamma_b = \frac{\omega}{\phi}$, and $\lambda = \frac{(1-\omega)(1-\theta)(1-\theta\beta)}{\phi}$.

Real marginal costs, mc^r , are difficult to measure, though. An early approach was to proxy them by using the (stationary) deviation of output from a linear/quadratic trend, or a HP-filtered series. Alternatively, Galí and Gertler (1999) and others argued in favor of proxying real marginal costs by average real unit labor costs. Under the special case of a (unitary substitution elasticity) Cobb–Douglas production function, real marginal costs reduce to the labor share; this has tended to be a common (if not the default) choice in the literature.⁷ If the elasticity of substitution between capital and labor is not unitary, however, such a proxy can lead to biased estimates.

In the following application, we estimate both NKPC forms (specifications (A.9) and (A.10)) over 1960q1-2012q4; the start of the sample is chosen for comparisons with the Gali-Gertler study. Note that the driving variable, i.e., the $\lambda(\cdot)$ term, whether it contains the output gap or the labor share, should, as befits a (price) gap term, be stationary. Stationarity in this context is simply another way of saying that there is co-integration between the optimal and actual price: $p_t^* - p_t$. In the case of a typical non-structural output gap measure that stationarity is assured. As we know, this is less clear for the labor share measures. For instance, revisiting **Figure 3**, we see (from the 1960s onwards) that **SE-GDP** and **PI₂-GDP** have exhibited a clear downward trend. The other three series are only borderline stationary in this period. This has a bearing on the success of the resulting estimates.

⁷Although see McAdam and Willman (2013b) for a derivation of real marginal costs in the NKPC framework assuming a CES production function and parametric factor utilization margins.

Outwardly, though, the NKPC estimations work relatively well across labor share types: parameters are correctly signed and tend to be significant (**Table A.4**). For example, $\hat{\beta}$ tends to be around the benchmark region of unity⁸. However, estimates of the duration of price fixedness vary from 8.5 – 13.8 quarters. Although these durations are high (compared, say, to micro price-setting evidence) they are by no means untypical in the literature (see the excellent survey by Mavroeidis et al. (2014)).⁹

Table A.4: New Keynesian Phillips Curve Estimates

	Naive-GDP	PI-GDP	PI₂-GDP	SE-GDP	BLS
SPECIFICATION (8)					
θ	0.891***	0.907***	0.928***	0.915***	0.909***
β	0.980***	1.004***	1.009***	1.009***	1.008***
λ	0.015***	0.009**	0.005	0.007**	0.008***
\mathcal{D}	9.2	10.7	13.8	11.8	11.0
SPECIFICATION (9)					
ω	0.104**	0.086**	0.089	0.065	0.035
θ	0.883***	0.901***	0.924***	0.912***	0.910***
β	0.961***	0.996***	1.002***	1.002***	1.004***
γ_b	0.106**	0.087**	0.088	0.066	0.037
γ_f	0.863***	0.909***	0.914***	0.936***	0.967***
λ	0.016***	0.009**	0.005	0.007**	0.008***
\mathcal{D}	8.5	10.1	13.1	11.4	11.1

Note: The covariance matrix was estimated with a 12 lags Newey-West estimator. The list of instruments is the same as in Galí and Gertler (1999): four lags of inflation, the labor share, the output gap, the long-short interest rate spread, wage and commodity price inflation. Galí (2015) additionally writes the NKPC instead using that $\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta} \cdot \Xi$ where $\Xi = \frac{1-\overline{LS}}{1-\overline{LS}+LS\epsilon}$ and \overline{LS} is the mean labor share and ϵ is the elasticity of substitution between product varieties. Using this formulation leads to a more reasonable price duration.

The slope parameters are of more interest here. To repeat, even though the driving variable should be stationary, at best our labor share series are borderline stationary. Accordingly, the minimization in the estimation algorithm places unusually low weights on the driving variable ($\lambda \in [0.005, 0.016]$). As predicted earlier, the **PI₂-GDP** and **SE-GDP** variants fare particularly poorly in that regard: the former never supports a statistically significant slope parameter, the latter supports a significant but quantitatively small one. Moreover, both of these specifications produce the most unreasonable price setting durations. The **NaiveGDP** and **PI-GDP** variants, by

⁸Occasionally, as in other studies, its point estimate numerically exceeds one marginally (indeed some authors set $\beta = 1$ in estimation for simplicity, Martins and Gabriel (2009)) but is still insignificantly different from standard values 0.95 – 0.99)

⁹For example, Galí et al. (2001), Gagnon and Khan (2005) and Smets and Wouters (2003) for the euro area.

contrast, have the lowest durations, significant slopes and significant parameters across both NKPC forms.

NKPCs are not, naturally, a fool-proof way of gauging inflation movements; there are other modelling approaches. That is not the main issue, though: our main point was that the NKPC literature gave a central explanatory role to the labor share of income. However, arguably this is not what most NKPC papers discuss. Much of the literature has instead become concerned with estimation and identification of dynamics (i.e., how much forward and backward-looking price setting there is), which are the best instruments to use in the GMM estimation, etc. The question of whether results are sensitive to which labor share measure we use has received little attention. In our case, though, we have highlighted that we can tie the success of NKPC estimation to the relative properties of the available labor share variants.

B Coherence

Table B.1 and **Table B.2** present (squared) coherence estimates.¹⁰ Keeping in mind that the annual numbers may be somewhat less reliable due to fewer observations, we find that coherence is always significant in the high-frequency domain. This result corroborates the previously formulated conclusion that labor share series tend to be rather consistent in the short run. Coherence estimates are more ambiguous in the lower frequencies, though. In the medium- and low frequency domain we identify subgroups for which coherence is very high, reflecting their definitional similarity: annual and quarterly series adjusted by proprietors' income (**PI-GDP** and **PI₂-GDP**), all "Naive" annual labor share series, all annual series adjusted for self-employment, and a pair consisting of quarterly series **SE-GDP** and **BLS**. Otherwise, the coherence is rather low.

The spectral analysis thus usefully highlights the discrepancies between various labor share series. From the frequency domain perspective it appears that the most outlying are the series adjusted for self-employment, which are characterized by substantially different variance decompositions and insignificant coherence with

¹⁰The (squared) coherence statistic for a pair of time series (x_t, y_t) can be understood as the R^2 from x_t regressed on y_t as a function of the frequency. Complementarily, one can also compute *dynamic correlation* coefficients, to control the sign of the relationship in each given pair. For all the pairs, dynamic correlation is in line with the general intuition, though: it is positive and significant whenever the coherence for a given pair is significant. In the case of insignificant coherence, the dynamic correlation is not significantly different from zero.

other variants in the low- and medium-run frequency. For example, (our favored series) **PI₂-GDP** is generally incoherent with the **SE-GVA** measure, considered the “headline measure” (and subsequently criticized) by Elsby et al. (2013).

Table B.1: Average Coherence Among the Labor Share Series – Annual Data

Periodicity	PI-GDP			PI ₂ -GDP			SE-GDP			Naive-GVA			Naive-GVA-NF			SE-GVA-NF			SE-GVA		
	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8
	DEMEANED SERIES																				
Naive-GDP	0.52***	0.67***	<u>0.73</u> ***	0.09	0.42**	<u>0.72</u> ***	0.47	0.19	<u>0.43</u> **	<u>0.86</u> ***	0.65***	0.77***	<u>0.89</u> ***	0.63***	0.54***	<u>0.42</u> **	0.15	0.31*	<u>0.48</u> ***	0.17	0.34*
PI-GDP				0.53***	<u>0.63</u> ***	0.63***	0.23	0.10	<u>0.48</u> ***	0.26	0.31*	<u>0.62</u> ***	0.33**	0.43**	<u>0.71</u> ***	0.04	0.22	<u>0.56</u> ***	<u>0.61</u> ***	0.22	0.43**
PI ₂ -GDP							0.05	0.20	<u>0.53</u> ***	0.01	0.14	<u>0.46</u> ***	0.01	0.19	<u>0.36</u> **	0.03	0.05	<u>0.27</u> *	0.07	0.07	<u>0.37</u> **
SE-GDP										<u>0.65</u> ***	0.26	0.35**	<u>0.57</u> ***	0.26	0.41**	0.11	0.33**	<u>0.52</u> ***	0.70***	0.78***	<u>0.85</u> ***
Naive-GVA													<u>0.98</u> ***	0.88***	0.81***	0.58***	0.38**	<u>0.58</u> ***	0.37**	0.20	<u>0.46</u> ***
Naive-GVA-NF																0.64***	0.62***	<u>0.87</u> ***	0.34**	0.29	<u>0.55</u> ***
SE-GVA-NF																			0.02	0.61***	<u>0.74</u> ***
	DE-TRENDED SERIES																				
Naive-GDP	0.00	0.42**	<u>0.68</u> ***	0.16	0.49***	<u>0.56</u> ***	0.46***	0.54***	<u>0.78</u> ***	0.37**	0.46***	<u>0.66</u> ***	0.41**	<u>0.51</u> ***	0.50***	0.19	0.26	<u>0.42</u> **	0.20	0.28	<u>0.66</u> ***
PI-GDP				<u>0.60</u> ***	0.60***	0.46***	0.31*	0.11	<u>0.54</u> ***	0.13	0.11	<u>0.56</u> ***	0.04	0.24	<u>0.66</u> ***	0.29*	0.21	<u>0.61</u> ***	0.49***	0.16	<u>0.53</u> ***
PI ₂ -GDP							0.01	0.26	<u>0.66</u>	0.04	0.11	<u>0.25</u>	0.01	0.12	<u>0.21</u>	0.13	0.05	<u>0.25</u>	0.15	0.11	<u>0.52</u> ***
SE-GDP										0.33**	<u>0.69</u> ***	0.56***	0.30*	<u>0.66</u> ***	0.44**	0.49***	<u>0.65</u> ***	0.50***	0.70***	0.81***	<u>0.91</u> ***
Naive-GVA													0.91**	<u>0.91</u> ***	0.83***	0.79***	<u>0.84</u> ***	0.78***	0.68***	<u>0.81</u> ***	0.72***
Naive-GVA-NF																0.79***	0.88***	<u>0.94</u> ***	0.55***	<u>0.77</u> ***	0.60***
SE-GVA-NF																			0.86***	<u>0.91</u> ***	0.71***

Note: ***, ** and * denote rejection of the null of coherence insignificance at 1%, 5% and 10% significance level, respectively. The spectra for a given pair have been estimated using the Parzen kernel.

Table B.2: Average Coherence Among the Labor Share Series – Quarterly Data

Periodicity	PI-GDP			PI ₂ -GDP			SE-GDP			BLS		
	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8
	DEMEANED SERIES											
Naive-GDP	0.33***	0.39***	<u>0.68</u> ***	0.21**	0.26**	<u>0.38</u> ***	0.01	0.01	<u>0.38</u> ***	0.04	0.08	<u>0.43</u> ***
PI-GDP				<u>0.82</u> ***	0.81***	0.51***	0.30***	0.32***	<u>0.69</u> ***	0.44***	0.48***	<u>0.81</u> ***
PI ₂ -GDP							<u>0.55</u> ***	0.54***	0.50***	<u>0.64</u> ***	0.62***	0.52***
SE-GDP										0.89***	0.88***	<u>0.89</u> ***
	DE-TRENDED SERIES											
Naive-GDP	0.78***	0.79***	<u>0.85</u> ***	0.64***	<u>0.65</u> ***	0.44***	0.18*	0.38***	<u>0.89</u> ***	0.11	0.26**	<u>0.77</u> ***
PI-GDP				<u>0.83</u> ***	0.80***	0.46***	0.24**	0.41***	<u>0.82</u> ***	0.22**	0.36***	<u>0.87</u> ***
PI ₂ -GDP							0.12	0.25**	<u>0.41</u> ***	0.08	0.17*	<u>0.40</u> ***
SE-GDP										0.89***	<u>0.91</u> ***	0.85***

C Additional Tables

Table C.1: AR(1) Persistence: Annual Labor Share

	Naive-GDP	PI-GDP	PI ₂ -GDP	SE-GDP	Naive-GVA	Naive-GVA-NF	SE-GVA-NF	SE-GVA
	<u>m₁</u>							
ρ	0.908***	0.840***	0.840***	0.939***	0.944***	0.881***	0.751***	0.872***
	<u>m₂</u>							
ρ	0.909***	0.852***	0.814***	0.726***	0.869***	0.780***	0.740***	0.812***
β_1 10 ⁴	-0.002	-0.029	-0.101***	-0.257***	0.118	0.122*	0.030	-0.130*
	<u>m₃</u>							
ρ	0.728***	0.759***	0.739***	0.716***	0.779***	0.700***	0.738***	0.774***
β_1 10 ⁴	0.727***	0.353*	0.197	-0.440**	0.655***	0.595**	0.164	-0.504*
β_2 10 ⁴	-0.007***	-0.004*	-0.004*	0.002	-0.005**	-0.005**	-0.002	0.004
	<u>m₁⁺</u>							
ρ	0.511***	0.528***	0.620***	0.454***	0.417***	0.388***	0.420***	0.485***

Note: ***, ** and * denote the rejection of null of insignificance at the 1%, 5% and 10% significance level, respectively (bootstrapped standard errors used). The estimated constants are omitted for brevity.

Table C.2: AR(1) Persistence: Quarterly Labor Share

	Naive-GDP	PI-GDP	PI ₂ -GDP	SE-GDP	BLS
	<u>m₁</u>				
ρ	0.968***	0.929***	0.975***	0.982***	0.964***
	<u>m₂</u>				
ρ	0.971***	0.920***	0.958***	0.864***	0.913***
$\beta_1 \cdot 10^4$	-0.002	-0.004	-0.006	-0.033***	-2.906***
	<u>m₃</u>				
ρ	0.884***	0.890***	0.920***	0.823***	0.843***
$\beta_1 \cdot 10^4$	0.083***	0.032**	0.033**	-0.004	8.095**
$\beta_2 \cdot 10^4$	-0.003***	-0.001**	-0.002***	-0.001**	-0.496***
	<u>m₁⁺</u>				
ρ	0.670***	0.627***	0.722***	0.633***	0.655***

Note: See Table C.1.

Table C.3: AR(1) and SV-AR(1) Models, Quarterly Labor Share Series

	naiveGDP	PI-GDP	PI ₂ -GDP	SE-GDP	BLS
ρ	0.670	0.627	0.722	0.633	0.655
	{0.581 : 0.759}	{0.528 : 0.726}	{0.631 : 0.814}	{0.539 : 0.726}	{0.562 : 0.749}
$\rho_{\tilde{l}_s}$	0.705	0.654	0.746	0.685	0.690
	{0.626 : 0.781}	{0.578 : 0.732}	{0.673 : 0.820}	{0.602 : 0.765}	{0.614 : 0.766}
ρ_σ	0.717	0.693	0.806	0.733	0.737
	{0.513 : 0.885}	{0.473 : 0.886}	{0.637 : 0.967}	{0.550 : 0.889}	{0.553 : 0.903}
$\bar{\sigma}$	-5.220	-5.152	-5.312	-5.167	-4.916
	{-5.368 : -5.075}	{-5.292 : -5.003}	{-5.504 : -5.054}	{-5.322 : -5.003}	{-5.066 : -4.754}
η_σ	0.287	0.293	0.259	0.282	0.281
	{0.206 : 0.376}	{0.211 : 0.385}	{0.179 : 0.349}	{0.200 : 0.367}	{0.202 : 0.370}

Note: The 95% AR bootstrapped confidence bands used. For the SV-AR(1) the 95% confidence intervals are given below the median estimates.

Table C.4: Share of Specific Frequencies in the Observed Variance (In %)

Periodicity (in years)	≥ 50	8-50	≤ 8
	Finland		
excluding the mean	79.5	16.5	4.0
excluding a linear trend	15.3	72.1	12.6
excluding a quadratic trend	12.9	73.4	12.7
	UK		
excluding the mean	66.3	25.9	7.9
excluding a linear trend	42.0	45.1	12.9
excluding a quadratic trend	41.5	45.5	13.0
	France		
excluding the mean	35.6	49.9	14.5
excluding a linear trend	16.9	65.9	17.2
excluding a quadratic trend	14.0	68.2	17.8

Note: the shares have been calculated using periodogram estimates. **Bold** indicates maximum value.

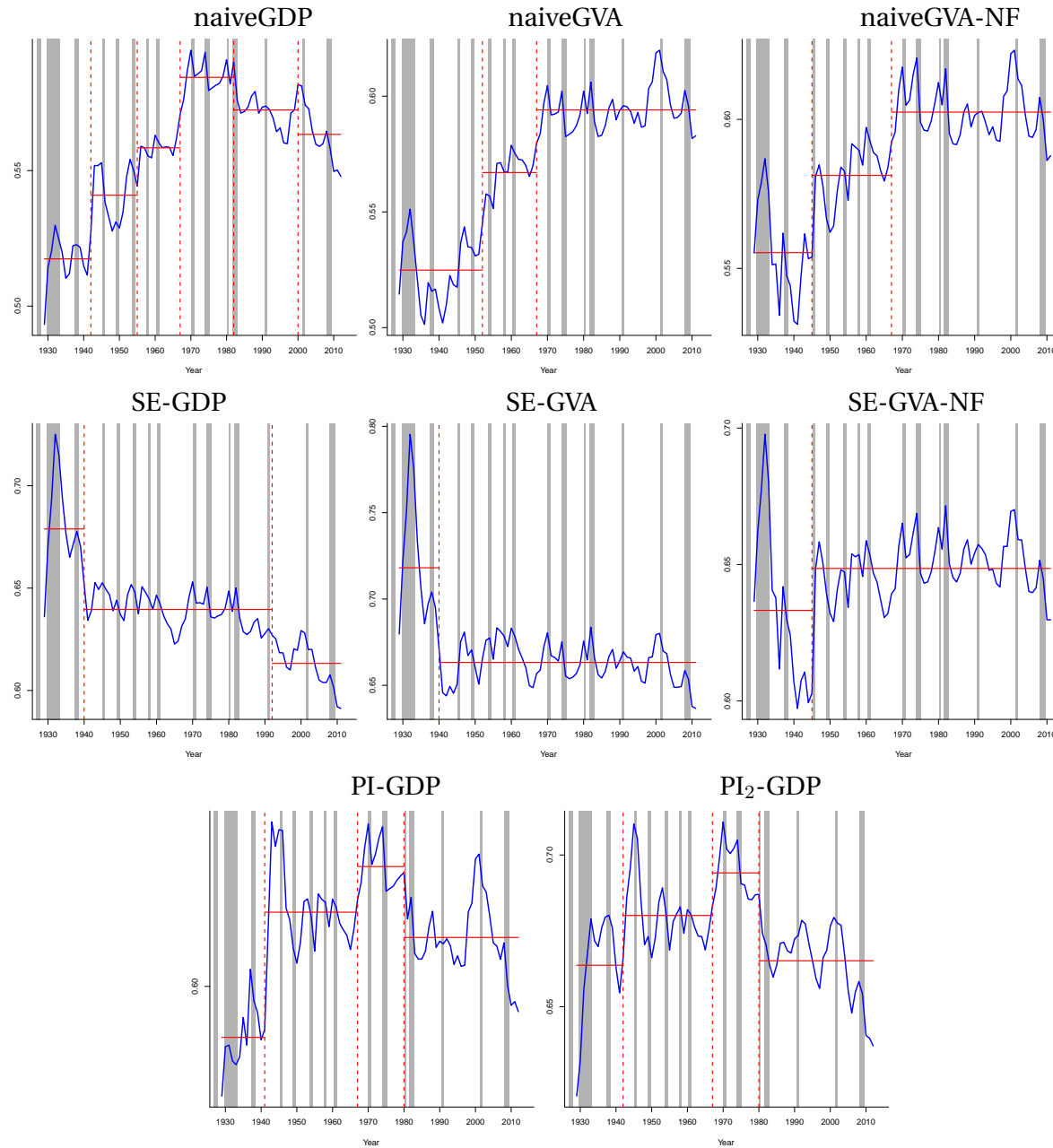
Table C.5: Detailed Description of Data Construction

Abbreviation	Description	Eqn	Freq.
Naive-GDP	Naive method, where CE_t : Compensation of Employees [Table 1.12 NIPA] and Y_t : GDP [Table 1.1.5 NIPA]	1	A&Q
SE-GDP	adjustment by self-employed, where Y_t is GDP [Table 1.1.5 NIPA], SE_t : self-employment in private economy [Table 6.7 NIPA] and TE_t is the sum of self-employment excluded and Full-Time Equivalent Employee [Table 6.5 NIPA]. In order to construct the quarterly labor share we use the date from BLS: Total Employment (sum of private [BLS CES0500000001 Series] and government [BLS CES9000000001 Series]) and Self-employment (sum of non-agriculture [BLS LNS12032192 Series] and agriculture self-employment [BLS LNS12032185 Series]).	2	A&Q
PI-GDP	Adjustment by proprietor's income, where CE_t : Compensation of Employees [Table 1.12 NIPA] and Y_t : GDP [Table 1.1.5 NIPA] and PI_t : Proprietors' income with IVA and CCAdj [Table 1.12 NIPA].	3	A&Q
PI₂ – GDP	Extended adjustment by proprietor's income (see Gomme and Rupert (2007)). Most of the time series were taken from [Table 1.12 NIPA], apart from the GDP [Table 1.1.5 NIPA] and Consumption of fixed capital [Table 1.7.5 NIPA]	4	A&Q
Naive-GVA	Naive method calculated for private sector, where CE_t : Compensation of Employees in private sector [Table 1.12 NIPA] and Y_t : GVA in private sector [Table 1.3.5 NIPA]	1	A
Naive-GVA-NF	Naive method calculated for non-farm private sector, where CE_t : Compensation of Employees in private sector [Table 1.12 NIPA] reduced by the CE for farms [Table 1.12 NIPA] and Y_t : GVA in private sector deduced by farms [Table 1.3.5 NIPA]	1	A
SE-GVA-NF	Adjustment by self-employed, where Y_t is GVA for private economy [Table 1.3.5 NIPA] reduced by GVA in farm sector [Table 1.3.5 NIPA], SE_t : self-employment in private economy reduced by farms [Table 6.7 NIPA] and TE_t is the sum of self-employment excluded by farms sector and Full-Time Equivalent Employee in private sector [Table 6.5 NIPA]	2	A
SE-GVA	Adjustment by self-employed, where Y_t is GVA for private economy [Table 1.3.5 NIPA], SE_t : self-employment in private economy [Table 6.7 NIPA] and TE_t is the sum of self-employment and Full-Time Equivalent Employee in private sector [Table 6.5 NIPA]	2	A
BLS	Labor Share in non-farm business sector [PRS85006173], 2005=100	-	Q

Note: All the variables except self-employed data are expressed in current USD. "A" = Annual, "Q" = quarterly frequencies.

D Additional Graphs

Figure D.1: Structural Breaks Detected with the Bai and Perron (2003) Procedure – Annual Series



E Indicative Sectoral Analysis

One of the hypothesized explanations for the labor share decline since 1970s pertains to changes in the sectoral structure of the US economy. As argued, e.g., by Elsby et al. (2013), sectors are subject to various degrees of cross-border integration, and recent decades have witnessed an enormous surge of globalization and offshoring. And when labor-intensive production moves to countries with lower labor costs, one could expect the aggregate labor share to go down. On the other hand, the simultaneous rise of the service sector, and financial services in particular, could have worked in the opposite direction.

However, as presented in **Table E.1** and **Figure E.1** based on World KLEMS data, factor shares have been far from constant at the sectoral level as well. For example, in Mining and quarrying [C] as well as various branches of manufacturing, the labor share has been systematically falling throughout the period 1947–2010, whereas in numerous other branches, and especially non-market service sectors such as Public administration, defence and compulsory social security [L], Education [M], and Health and social work [N], it has been systematically rising.

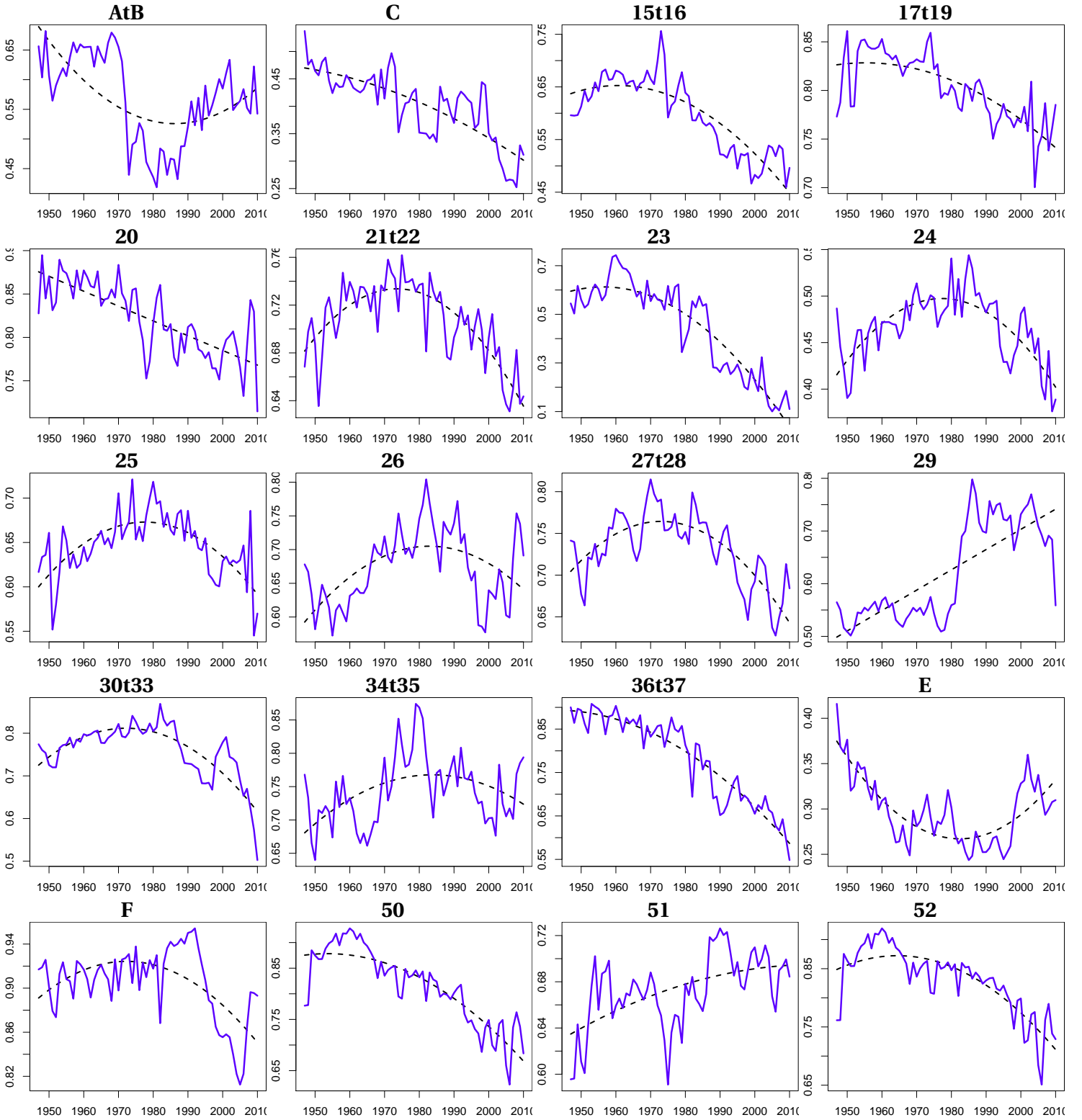
Hence, results of shift-share analyses – i.e., contributions of respective sectors to the total change in the aggregate labor share – are going to be driven both by the within- and between-sector component. **Figure E.2** illustrates this point. The shares of labor remuneration in manufacturing, as well as agriculture and mining, in total labor remuneration have been systematically falling throughout the entire period 1947–2010, driven both by declining labor shares in these sectors and their declining share of total value added. Market services provide a mirror image of this result. However, the financial sector, whose rise was hypothesized to be one of the drivers of labor share declines, provides a particularly interesting result here. In fact, its share in total labor remuneration has increased in the recent years, as the increase in its share of value added has outrun the labor share decline in this sector.

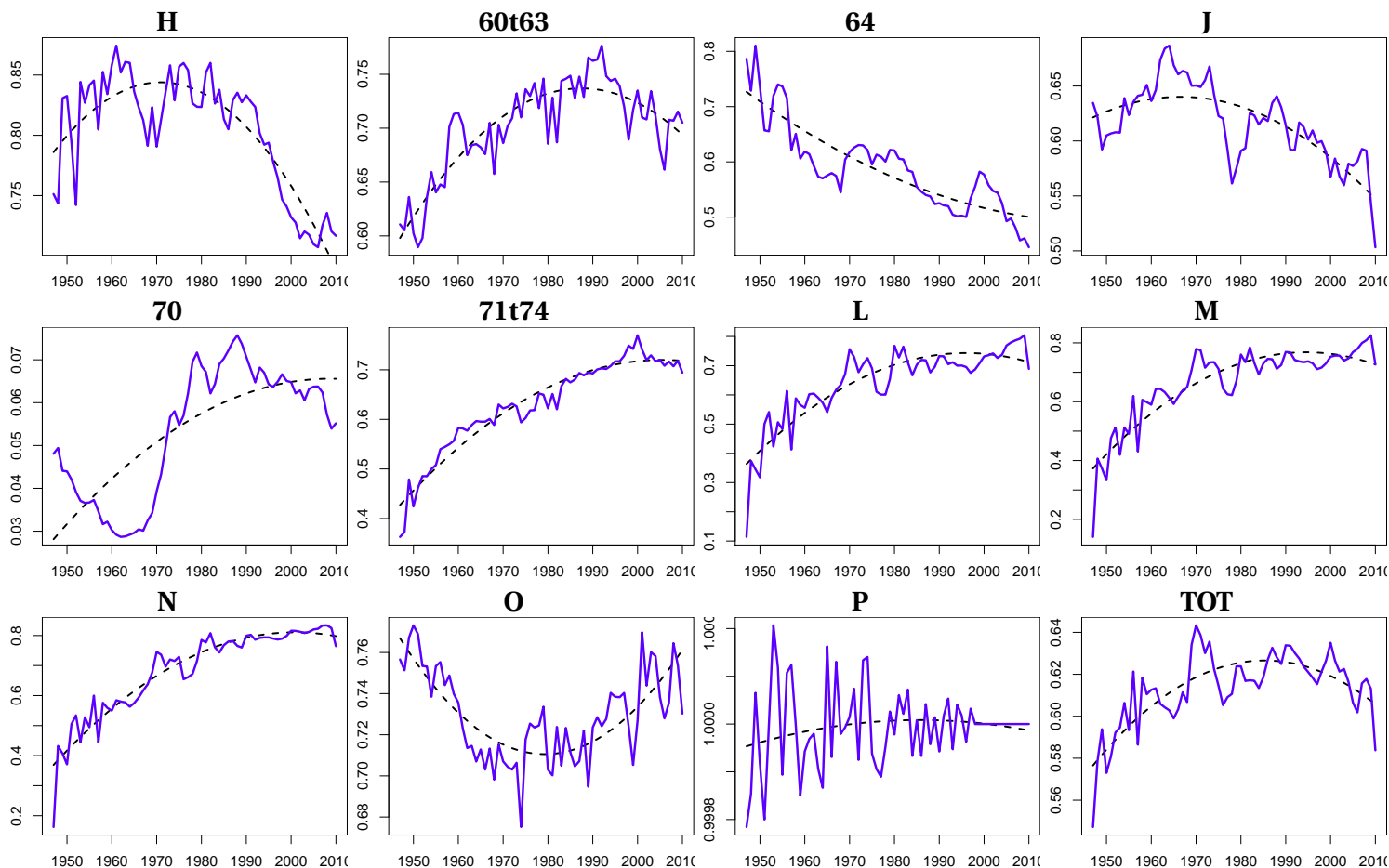
Table E.1: Share of US Sectors in Gross Value Added, Labor Share in Sectoral GVA, and Unit Root Tests

	share in value added ($w_{i,t}$)		labor share ($ls_{i,t}$)		ADF test		PP test	
	\bar{w}_i	$w_{i,2010} - w_{i,1947}$	\bar{ls}_i	$ls_{i,2010} - ls_{i,1947}$	const	trend	const	trend
AtB	3.24	-8.54	0.57	-0.11	-1.70	-1.72	-2.22	-2.27
15t16	2.16	-2.21	0.60	-0.10	-1.19	-3.25*	-1.28	-3.16*
17t19	1.43	-3.32	0.80	0.01	-2.25	-5.26***	-2.98**	-5.79***
20	0.83	-0.86	0.82	-0.11	-2.46	-5.32***	-2.53	-5.56***
21t22	2.02	-1.17	0.71	-0.02	-2.09	-2.85	-2.99**	-3.81**
23	0.55	0.51	0.45	-0.43	-0.60	-3.29*	-0.79	-3.26*
24	1.99	-0.08	0.47	-0.10	-1.76	-1.77	-2.58	-2.57
25	0.70	-0.25	0.65	-0.05	-2.14	-2.11	-4.01***	-3.99*
26	0.68	-0.61	0.67	0.01	-2.67*	-2.77	-2.60*	-2.72
27t28	3.06	-2.98	0.73	-0.06	-2.31	-2.76	-2.19	-2.58
29	1.83	-1.05	0.62	-0.01	-1.56	-1.30	-1.39	-1.26
30t33	2.32	0.27	0.76	-0.27	0.73	-0.15	0.98	0.07
34t35	2.58	-1.24	0.74	0.03	-2.67*	-2.83	-3.13**	-3.33*
36t37	0.56	0.00	0.78	-0.35	-0.01	-3.16*	-0.41	-3.87**
50	1.19	-0.29	0.81	-0.09	-1.28	-5.72***	-1.41	-4.80***
51	5.37	-1.11	0.67	0.09	-3.47***	-3.70**	-3.46***	-3.77**
52	5.05	-2.61	0.83	-0.03	-2.01	-5.56***	-2.21	-4.75***
60t63	4.01	-3.37	0.70	0.09	-2.43	-2.10	-2.60*	-2.77
64	2.82	0.93	0.59	-0.34	-1.44	-2.72	-2.02	-3.33*
70	10.75	3.00	0.05	0.01	-1.16	-1.78	-0.91	-1.12
71t74	7.94	11.56	0.63	0.33	-4.37***	-3.70*	-3.81***	-3.24*
C	1.96	-0.95	0.40	-0.23	-2.02	-3.61**	-2.63*	-4.24***
E	2.56	0.22	0.30	-0.11	-2.72*	-2.44	-3.58***	-3.19*
F	4.76	-0.36	0.90	-0.02	-2.00	-2.14	-2.39	-2.59*
H	2.29	0.11	0.80	-0.03	-1.41	-3.29*	-1.71	-3.18*
J	5.13	6.52	0.62	-0.13	-0.85	-2.16	-0.70	-1.80
L	5.42	-3.83	0.64	0.58	-3.14**	-3.55**	-4.81***	-5.96***
M	4.24	3.88	0.66	0.59	-2.99**	-3.08	-4.72***	-5.59***
N	8.53	8.27	0.68	0.60	-2.65*	-2.74	-4.33***	-6.24***
O	3.69	0.36	0.73	-0.03	-2.47	-2.41	-3.06**	-3.00
P	0.31	-0.79	1.00	0.00	-7.06***	-6.98***	-7.77***	-7.70***
TOT	100.00		0.61	0.04	-3.03***	-2.33	-4.29***	-3.74**

Note: ADF and PP stand for Augmented Dickey-Fuller and Phillips-Perron test, respectively. Super-scripts ***, ** and * denote the rejection of null about unit root at 1%, 5% and 10% significance level, respectively.

Figure E.1: Payroll share in the US sectors

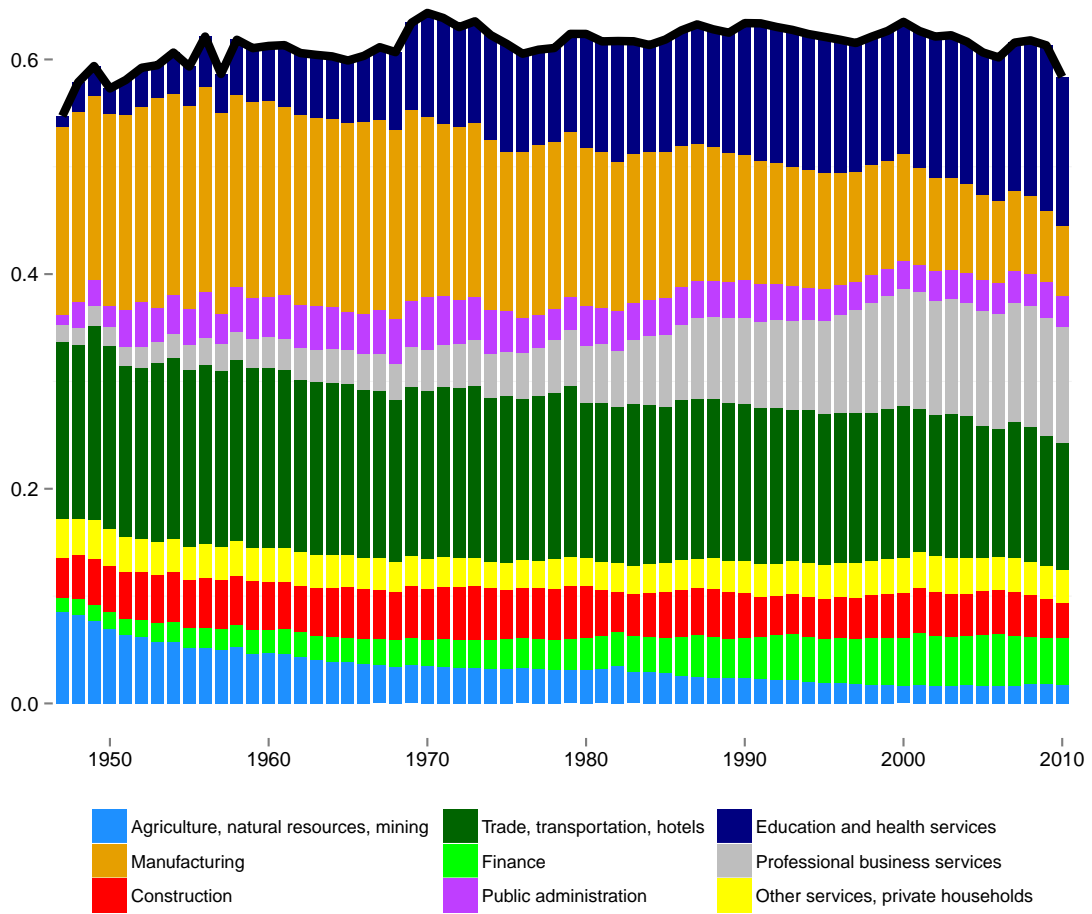




Note: solid and dashed lines stand for the payroll share in given sector and its long-run tendency, respectively.

Sectors: Total economy [TOT]; Agriculture, hunting, forestry and fishing [AtB]; Mining and quarrying [C]; Manufacture of food products, beverages and tobacco products [15t16]; Manufacture of textiles, wearing apparel, dressing and dyeing of fur, luggage, handbags, saddlery, harness and footwear and tanning and dressing of leather [17t19]; Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials [20]; Manufacture of paper and paper products, publishing, printing and reproduction of recorded media [21t22]; Manufacture of coke, refined petroleum products and nuclear fuel [23]; Manufacture of chemicals and chemical products [24]; Manufacture of rubber and plastics products [25]; Manufacture of other non-metallic mineral products [26]; Manufacture of basic metals, fabricated metal products, except machinery and equipment [27t28]; Manufacture of machinery and equipment n.e.c. [29]; Manufacture of office, accounting, computing machinery, electrical machinery, apparatus, radio, television, communication equipment, medical, precision and optical instruments, watches and clocks [30t33]; Manufacture of motor vehicles, trailers, semi-trailers and other transport equipment [34t35]; Manufacture of furniture, manufacturing n.e.c. and recycling [35t37]; Electricity, gas and water supply [E]; Construction [F]; Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of automotive fuel [50]; Wholesale trade and commission trade, except of motor vehicles and motorcycles [51]; Retail trade, except of motor vehicles and motorcycles; repair of personal and household goods [52]; Hotels and restaurants [H]; Land transport, transport via pipelines, water transport, air transport, supporting, auxiliary transport activities and activities of travel agencies [60t63]; Post and telecommunications [64]; Financial intermediation [J]; Real estate activities [70]; Renting of machinery and equipment without operator and of personal and household goods, computer and related activities, research and development and other business activities [71t74]; Public administration, defence and compulsory social security [L]; Education [M]; Health and social work [N]; Other community, social and personal service activities [O]; Private households with employed persons [P].

Figure E.2: Sectoral Decomposition of the Annual US Labor Share



F Unit Root Tests and Structural Breaks

Table F.1: Unit Roots Tests: Annual

	Naive-GDP	PI-GDP	PI ₂ -GDP	SE-GDP	Naive-GVA	Naive-GVA-NF	SE-GVA-NF	SE-GVA
ADF								
(1)	-2.10	-2.77**	-3.83***	-2.25	-1.45	-2.11	-3.73***	-3.72***
(2)	-1.69	-2.53	-4.36***	-4.35***	-2.37	-3.01	-3.93**	-4.19***
PP								
(1)	-2.82*	-3.13**	-3.42**	-1.68	-1.83	-2.35	-3.55***	-2.65*
(2)	-2.08	-2.80	-3.79**	-4.39***	-2.26	-3.03	-3.56**	-3.39*
ADF-GLS								
(1)	-0.58	-1.18	-1.23	-1.02	-0.56	-1.52	-3.31***	-2.37**
(2)	-1.07	-1.69	-1.61	-2.83*	-2.09	-3.32**	-3.86***	-3.56**
\mathcal{H}_0 : ESTAR								
(1)	-2.687*	-2.874*	-4.007***	-2.798*	-1.896	-2.695*	-3.576***	-3.423**
(2)	-2.013	-2.984	-4.256***	-3.634**	-3.307*	-3.426**	-3.329*	-3.804**
\mathcal{H}_0 : asymmetric ESTAR								
(1)	4.450*	5.248**	8.038***	4.275*	2.334	4.634*	6.439**	6.435**
(2)	2.516	4.476	9.190***	10.379***	6.232*	6.928**	6.032*	7.361**
Fractional								
\hat{d}	0.791	0.639	0.421	0.577	0.884	0.696	0.836	0.649
$\hat{d} = 0$	[0.072]	[0.015]	[0.560]	[0.114]	[0.006]	[0.003]	[0.047]	[0.011]
$\hat{d} = 1$	[0.328]	[0.091]	[0.007]	[0.048]	[0.588]	[0.154]	[0.448]	[0.101]

Note: ***, ** and * denote rejection of the null hypothesis of a unit root for all tests at the 1%, 5% and 10% significance level, respectively. Models (1) and (2) incorporate only a constant and a constant and deterministic trend, respectively. The ADF ESTAR and asymmetric ADF ESTAR follow Kapetanios et al. (2003) and Sollis (2009), respectively.

Table E.2: Number of Breaks with Corresponding Breakpoints – Annual Series

	Naive-GDP	PI-GDP	PI₂-GDP	SE-GDP	Naive-GVA	Naive-GVA-NF	SE-GVA-NF	SE-GVA
	WITH MEAN							
1940-1950s	1942 (1940, 1944) 1955 (1954, 1959)	1941 (1940, 1942)	1942 (1938, 1955)	1940 (1939, 1945)		1945 (1943, 1949)	1945 (1942, 1987)	1940 (1939, 1946)
1960-1970s	1967 (1966, 1968)	1967 (1964, 1976)	1967 (1963, 1974)		1952 (1951, 1954) 1967 (1965, 1968)	1967 (1964, 1970)		
1980-1990s	1982 (1980, 1984)	1980 (1977, 1981)	1980 (1978, 1981)	1992 (1990, 1993)				
2000s	2000 (1987, 2004)							
	WITH LINEAR TREND							
1940-1950s		1942 (1939, 1943) 1955 (1954, 1968)	1946 (1945, 1956)	1940 (1938, 1950)	1942 (1941, 1945)	1941 (1940, 1946)	1945 (1944, 1951)	1945 (1944, 1952)
1960-1970s	1968 (1967, 1969)	1968 (1967, 1969)	1968 (1967, 1969)	1968 (1962, 1969)	1974 (1973, 1975)	1968 (1966, 1969)		
1980-1990s		1982 (1977, 1983)	1985 (1984, 1987)					
2000s	1999 (1997, 2000)	1999 (1998, 2000)	1999 (1996, 2000)		1999 (1996, 2000)	1997 (1994, 1998)	1999 (1997, 2000)	

Note: The breakpoints are calculated in two steps. In the first step, we estimate all the possible models with a number of structural breaks varying from 1 to 5. In the second step, we choose one with the lowest BIC criterion. The years in parentheses are 95% confidence intervals.

Table F.3: Number of Breaks with Corresponding Breakpoints – Quarterly Series

	Naive-GDP	PI-GDP	PI₂-GDP	SE-GDP	BLS
WITH MEAN					
1940-1950s	1956q3 (1956q2, 1957q3)				
1960-1970s	1967q3 (1967q2, 1967q4)	1968q2 (1967q3, 1968q4)	1967q4 (1967q1, 1968q2)	1962q2 (1961q3, 1962q4)	
1980-1990s	1983q1 (1982q2, 1983q4)	1980q4 (1979q4, 1981q2)	1980q3 (1980q2, 1981q1)	1983q2 (1983q1, 1984q2)	1983q1 (1982q3, 1985q4)
				1993q3 (1992q2, 1993q4)	1992q4 (1989q4, 1993q3)
2000s	2003q2 (2002q3, 2003q4)	2003q2 (2000q2, 2005q4)	2003q2 (2002q1, 2003q4)	2003q2 (2002q3, 2003q4)	2003q2 (2002q3, 2003q3)
WITH LINEAR TREND					
1940-1950s	1958q1 (1957q4, 1959q4)			1957q3 (1957q1, 1958q1)	1956q3 (1955q2, 1957q1)
1960-1970s	1968q2 (1967q3, 1968q3)	1968q2 (1967q3, 1968q3)	1969q1 (1968q3, 1969q2)	1969q1 (1968q4, 1969q3)	1969q1 (1968q4, 1969q3)
				1978q4 (1978q1, 1979q4)	1978q4 (1978q2, 1980q1)
1980-1990s	1983q1 (1982q3, 1985q4)	1983q1 (1981q3, 1983q2)	1986q1 (1985q4, 1986q3)		
	1999q4 (1998q4, 2000q1)	1999q4 (1999q3, 2000q1)	1999q4 (1998q4, 2000q1)	1999q4 (1998q4, 200q1)	1999q4 (1998q4, 200q1)
WITH QUADRATIC TREND					
1940-1950s					
1960-1970s	1960q3 (1960q2, 1960q4)	1960q3 (1960q2, 1960q4)	1960q1 (1959q4, 1960q2)	1960q3 (1960q2, 1960q4)	1960q3 (1960q2, 1960q4)
	1970q2 (197q1, 1970q3)	1970q2 (197q1, 1970q3)	1969q4 (1969q3, 1970q1)	1970q2 (197q1, 1970q3)	1970q2 (197q1, 1970q3)
1980-1990s	1983q1 (1982q4, 1983q2)	1986q3 (1986q2, 1986q4)	1981q2 (1980q4, 1981q4)	1983q1 (1982q4, 1983q2)	1983q1 (1982q4, 1983q2)
	1997q4 (1997q3, 1998q1)	1999q4 (1999q3, 200q1)	1992q1 (1991q4, 1992q2)	1997q4 (1997q3, 1998q1)	1997q4 (1997q3, 1998q1)
2000s			2001q4 (2001q3, 2002q1)		

Note: The breakpoints are calculated in two steps. In the first step, we estimate all the possible models with a number of structural breaks varying from 1 to 5. In the second step, we choose one with the lowest BIC criterion. The years in parentheses are confidence intervals at 95% significance level.

Table F4: Zivot and Andrews (1992) Test for a Unit Root Subject to a Structural Break – Annual Series

	intercept		trend		intercept and trend	
	τ	\mathcal{B}	τ	\mathcal{B}	τ	\mathcal{B}
Naive-GDP	-3.31	1952	-4.44**	1975	-5.12**	1967
PI-GDP	-4.59*	1942	-3.62	1944	-4.48	1942
PI ₂ -GDP	-4.44	2004	-4.35*	2002	-4.63	1967
SE-GDP	-6.33***	1935	-6.30***	1941	-6.09***	1944
Naive-GVA	-3.87	1952	-3.36	1971	-3.86	1952
Naive-GVA-NF	-3.98	1952	-3.93	1974	-4.61	1942
SE-GVA-NF	-5.08***	1934	-4.73**	1936	-6.50***	1946
SE-GVA	-7.27***	1934	-6.73***	1941	-6.85***	1945

Note: τ and \mathcal{B} denote the test statistic in Zivot-Andrews procedure and its break-point, respectively. Asterisks ***, ** and * denote rejection of the null about unit root at 1%, 5% and 10% significance level, respectively.

Table F5: Zivot and Andrews (1992) Test for a Unit Root Subject to a Structural Break – Quarterly Series

	intercept		trend		intercept and trend	
	τ	\mathcal{B}	τ	\mathcal{B}	τ	\mathcal{B}
Naive-GDP	-3.74	1966q2	-4.58**	1972q1	-5.03*	1968q3
PI-GDP	-4.32	1966q2	-4.30*	1970q1	-4.53	1974q4
PI ₂ -GDP	-3.74	1966q2	-3.81	1970q2	-4.13	1966q2
SE-GDP	-5.28***	1967q3	-5.26***	2000q2	-5.47**	1999q1
BLS	-4.79**	2009q1	-5.05***	2000q4	-5.11**	2000q1

Note: As in table F4.

The key message from the unit root tests for the log-differences between the labor share variants is that most of the proposed adjustments to the Naive calculation are not constant over time. Please recall that there are two key tendencies which are responsible for that facts (cf. **Figure 1**): (i) a downward trend in the ratio of the self-employed to employees, and (ii) a systematic decrease in the share of ambiguous income in total output.