Online Appendix for

"Endogenous Labor Share Cycles: Theory and Evidence"

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A Data Construction

A.1 Labor Share

The broadly used approach in measuring the labor share is simply dividing Compensation of Employees (CE) by GDP. But that does not take incomes of the self-employed into consideration. Unfortunately, labor income of the self-employed is published jointly with the capital income. Since Gollin (2002) a number of adjustments have been proposed. We incorporate one of the most detailed ways in the measuring the labor share, suggested by the seminal Gomme and Rupert (2007) paper, which takes into consideration the unknown (self-employed) income. The starting point is the assumption that the proportion of the unknown labor (capital) income to the total unknown income is the same as the ratio of known labor (capital) to the total known income. The unknown income (AI) is the sum of Proprietors' Income (PI), Business Current Transfer Payments ($BCTP$), Statistical Discrepancy ($SDis$) and Taxes on Production (Tax) reduced by Subsidies (Sub) ($AI = PI + Tax - Sub + BCTP + SDis$). On the other hand, known capital income (UCI) consists of Rental Income (RI), Current Surplus of Government Enterprises (GE), Net Interests (NI) and Corporate Profits (CP). Including UCI to Compensation of Employees (CE) and consumption of fixed capital (DEP) we derive total unambiguous income (UI) and can calculate the portion of UCI to UI: $\kappa = \frac{UCI + DEP}{UI}$. Having κ, it is easy to obtain ambiguous capital income (ACI) which equals $AC \cdot \kappa$. Finally, we derive labor share income as one minus capital income share:

$$LS = 1 - \frac{UCI + DEP + ACI}{GDP} = 1 - \kappa.$$

GDP and Consumption of fixed capital (DEP) are taken from NIPA *[Table 1.7.5]* and the remaining series are taken from NIPA *[Table 1.12]*.

A.2 Macroeconomic Variables

GDP - Gross Domestic Product in billions of chained (2005) dollars, BEA NIPA Table 1.6.
Capital stock to product (K_t/Y_t) - ratio of non-residential private fixed assets stock in current billions of dollars to GDP also in current billions of dollars, series taken from BEA NIPA Table 1.5 and BEA Fixed Assets Table 4.1, respectively.
Consumption (C_t) - consumption of non-durable goods and services in current billions of dollars deflated by the implicit GDP deflator, series taken from BEA NIPA Table 1.5 and 1.6.
Investment (I_t) - sum of non-residential private fixed investment and consumption of durable goods in current billions of dollars deflated by the implicit GDP deflator, series taken from BEA NIPA Table 1.5 and 1.6.
Consumption to product ratio (C_t/Y_t) - ratio of consumption of non-durable goods and services in current billions of dollars to GDP also in current billions of dollars, series taken from BEA NIPA Table 1.5.
Investment to product ratio (I_t/Y_t) - ratio of consumption of durable goods and non-residential private fixed investment in current billions of dollars to GDP also in current billions of dollars, series taken from BEA NIPA Table 1.5.

The share of the R&D expenditures in Gross Domestic Product (RD_t/GDP_t) is ratio of total spending on research and development sector divided by GDP, both series in current millions of dollars, taken from National Science Foundation and BEA NIPA Table 1.5, respectively.
The share of the non-federal R&D expenditures in Gross Domestic Product (RD_t^{NF}/GDP_t) is ratio of non-federal spending on research and development sector divided by GDP, both series in current millions of dollars, taken from National Science Foundation and BEA NIPA Table 1.5, respectively.
R&D expenditures (RD_t) – Research and development expenditures in constant millions of dollars, series taken from National Science Foundation.
Non-federal R&D expenditures (RD_t^{NF}) – Non-federal research and development expenditures in constant millions of dollars, series taken from National Science Foundation.
Skill premium (w_t^S/w_t^U) - composition adjusted college/high school log weekly wage ratio, series taken from Acemoglu and Autor (2011).
Labor productivity ($LaborProd_t$) - Real output per hour in non-farm business sector, index (2009=100), BLS Series No. OPHNFB.
Employment L_t - Employment in non-farm business sector, index (2009=100), BLS Series No. PRS85006013.
Aggregate hours $L_t \times h_t$ - Aggregate hours in non-farm business sector, index (2009=100), BLS Series No. HOANBS
Consumption to capital stock C_t/K_t - ratio of consumption of non-durable goods and services in current billions of dollars to non-residential private fixed assets stock also in current billions of dollars, series taken from BEA NIPA Table 1.5 and BEA Fixed Assets Table 4.1, respectively.

B Some Simple Time Series Properties for the US Labor Share

B.1 Descriptive Statistics

Table B.1: Labor Share: Summary Statistics

	Annual	Quarterly
Mean	0.655	0.649
Max	0.706	0.691
Min	0.607	0.600
Std. Dev.	0.021	0.020
Obs.	87	276
	(1929-2015)	(1947:1-2015:4)

B.2 Persistence and Cyclicality

To scrutinize the persistence of the labor share, we assume that it follows an auto-regressive process:

$$y_t = \mu + \rho_y y_{t-1} + error_t,$$

where the drift term μ captures the long-run mean, $\mu/(1-\rho_y), \rho_y \neq 1$. Our interest focuses on the value of ρ_y (the persistence parameter). **Table B.2** demonstrates that the labor share is a highly persistent, slowly adjusting series (with ρ_y around 0.8-0.9 and over 0.9 for annual and quarterly series, respectively). As can be seen, these high persistence values are robust to the inclusion of a linear or quadratic trend.[34]

Finally **Table B.3** shows the counter-cyclicality of the raw labor share data using regression on a recession dummy, $NBER$.

Table B.2: AR(1) Model Estimates for the Labor Share

	Annual			Quarterly		
	(1)	(2)	(3)	(1)	(2)	(3)
	(1)	(2)	(3)	(1)	(2)	(3)
$\hat{\rho_y}$	0.924***	0.799***	0.761***	0.977***	0.940***	0.927***
$\rho_y = 1$	[0.072]	[0.000]	[0.000]	[0.056]	[0.004]	[0.001]

Note: Superscripts ***, ** and * denote the rejection of null about parameter's insignificance at 1%, 5% and 10% significance level, respectively. Probability values in squared brackets.
Specifications:
(1): $y_t = \mu + \rho_y y_{t-1} + error_t$
(2): $y_t = \mu + \rho_y y_{t-1} + \beta_1 t + error_t$
(3): $y_t = \mu + \rho_y y_{t-1} + \beta_1 t + \beta_2 t^2 + error_t$

Table B.3: Counter-Cyclicality of Labor Share Series

	(1)	(2)	(3)	(4)
Annual				
$\hat{\mathcal{D}}$	0.00997**	0.00380**	0.00292*	0.00299*
Quarterly				
$\hat{\mathcal{D}}$	0.00299*	0.00092	0.00107*	0.00120*

Notes: $NBER = 1$ if the economy is in a recession as identified by the NBER chronology and 0 otherwise.
Specifications:
(1): $y_t = \mu + \mathcal{D} \times NBER_t + error_t$
(2): $y_t = \mu + \mathcal{D} \times NBER_t + \rho_y y_{t-1} + error_t$
(3): $y_t = \mu + \mathcal{D} \times NBER_t + \rho_y y_{t-1} + \beta_1 t + error_t$
(4): $y_t = \mu + \mathcal{D} \times NBER_t + \rho_y y_{t-1} + \beta_1 t + \beta_2 t^2 + error_t$.

[34]Although, naturally, these alternative forms relax the assumption about the uniqueness of the labor share's equilibrium level.

B.3 Stationarity

The next step consists in verifying stationarity of the labor share series. The regressions shown in **Table B.4** are performed in levels with an intercept. The results for the intercept-plus-trend case are available on request, although given our subject matter, the former case is more definitionally consistent. The tests are denoted as ADF = Augmented Dickey Fuller; ERS DF-GLS = Elliott-Rothenberg-Stock (1996), Dickey-Fuller GLS; PP = Philips-Perron; KPSS = Kwiatkowski-Phillips-Schmidt-Shin (1992); ERS = Elliott-Rothenberg-Stock (1996) point-optimal unit root; multiple Ng-Perron (2001) tests. Descriptions of these tests can be readily found in econometrics textbooks. The null in each case is that the series has a unit root (except for the KPSS test which has stationarity as the null). In each case the number of lags in the stationarity equation is determined by Schwartz Information criteria. In the Philips-Perron and KPSS methods, we use the Bartlett Kernel as the spectral estimation method and Newey-West bandwidth selection. ARFIMA models estimated by ML, with robust 95% confidence intervals given under the central estimate of fractional integration parameter $\hat{d}$.

Table B.4: Labor Share: Stationarity Tests

	Annual	$CV_{5\%}$	Quarterly	$CV_{5\%}$
ADF	[0.063]	–	[0.321]	–
ERS DF-GLS	−1.117	−1.940	−0.170	−1.940
PP	[0.368]	–	[0.292]	–
KPSS	0.816	0.463	1.401	0.463
ERS	4.480	3.070	24.400	3.200
Ng-Perron				
MZa	−2.010	−8.100	−0.287	−8.100
MZb	−0.980	−1.980	−0.159	−1.980
MSB	0.487	0.230	0.557	0.230
MPT	11.930	3.170	20.924	3.170
ARFIMA (0,d,0)	0.494		ARFIMA (0,d,0)	0.499
	(0.488,0.499)			(0.498,0.499)
ARFIMA (1,d,0)	0.439		ARFIMA (2,d,0)	0.191
	(0.374,0.505)			(-0.067,0.449)

Note: Squared brackets indicate probability values, $CV_{5\%}$ denotes the 5% critical value of the relevant test, and 95% confidence intervals for the ARFIMA differencing parameter are given in brackets.

B.4 Additional Results for Short-Run Labor Share Features

Here we look at two additional filtering methods for the short-run labor share series. These results are parallel to those of table 2 in the main text. We use three methods: the HP filter (as before) plus simple first differencing of the raw labor share series and filtering using CF. Results are relatively similar over the methods: the relative variance of the annual (quarterly) series is around $0.4(0.55)$, the degree of persistence is 0.3 (0.7 for the last two methods) and there is no sign of significant cyclicality (although the point estimates are negative).

Table B.5: Main features of the labor share's short-run component

	σ_{LS_t}	$\sigma_{LS_t}/\sigma_{GDP_t}$	$\rho_{LS_t,LS_{t-1}}$	ρ_{LS_t,GDP_t}
first-differenced				
annual series	0.986	0.453	0.282	-0.020
	(0.854, 1.122)		(0.028, 0.522)	(-0.284, 0.232)
quarterly series	0.588	0.658	-0.072	-0.292
	(0.511, 0.672)		(-0.246, 0.121)	(-0.458, -0.107)
HP-filtered				
annual series	0.660	0.486	0.325	-0.098
	(0.538, 0.771)		(0.135, 0.495)	(-0.303, 0.111)
quarterly series	0.785	0.513	0.736	-0.185
	(0.704, 0.870)		(0.673, 0.795)	(-0.289, -0.074)
CF-filtered (periodicity below 8 years)				
annual series	0.638	0.419	0.242	-0.021
	(0.516, 0.746)		(0.022, 0.433)	(-0.218, 0.171)
quarterly series				
	0.751	0.493	0.714	-0.133
	(0.668, 0.837)		(0.649, 0.776)	(-0.245, -0.020)

Note: σ_{LS_t} and $\sigma_{LS_t}/\sigma_{GDP_t}$ denotes volatility in absolute term (percentage deviation from the long-run trend) and relative term (as a ratio to the GDP's volatility). $\rho_{LS_t,LS_{t-1}}$ and ρ_{LS_t,GDP_t} stand for the first-order autocorrelation and contemporaneous comovement with product, respectively.

C Results for Other Countries

To strengthen the empirical points we make in Section 2, we shall argue that data for other developed economies support our main points as well. It turns out that, despite slight definitional changes, time-series properties of the labor share in Finland (1900-2003), the UK (1855-2010), and France (1896-2008) are broadly in line with our main set of findings for the US.

In order to explore the main features of the labor share in Finland we use the data set compiled by Jalava et al. (2006). At the first sight, the long-run trajectory of the Finnish labor share index appears markedly different in comparison to the US labor share (see figure C.1). However, if we consider only the postwar sample then a hump-shaped tendency is again well-identified, with the peak in the beginning of 1980s.

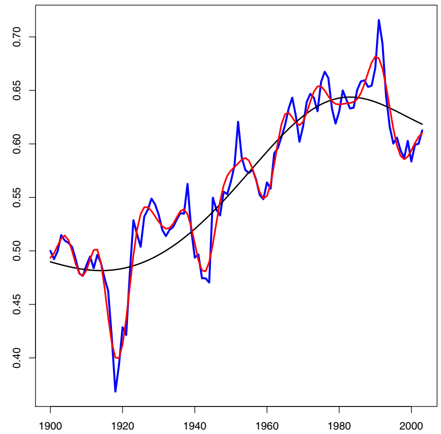
Our results for the UK and France are from Piketty (2014). The UK labor share (see figure C.2) exhibits substantial medium- and long-run variability and its general pattern since 1920s is very similar to the one of the US: a clear upward swing until around 1975, followed by a period of gradual decline. Importantly, for the UK we also observe gradual decline of the labor share in 1855–1916, in line with our interpretation that this variable can be subject to long cycles. Evidence for France (figure C.3) is less clear-cut (the data may be subject to a structural break in the 1940s).

Table C.6 shows the estimates of spectral density for the Finnish, British, and French labor share. We find that when the labor share is only demeaned, its volatility is dominated by low-frequency oscillations. De-trending the series by subtracting a linear or quadratic trend limits the importance of the cycles with lowest frequencies in favor of the medium-term component. In that case, medium-run fluctuations are responsible for more than 65 % of overall variance in Finland and France, and more than 45% in the UK. The latter result for the UK stems from the fact that the data period since 1855 allows us to identify more than one swing in the time series, which thus becomes badly fitted with any quadratic trend.

We conclude that the medium- and long-run swings are a very important not only for the US labor share, but also for few notable European ones.

Finally, we also confirm on the basis of Jalava et al. (2006) data that the medium-term component of the labor share in Finland has also been highly persistent and pro-cyclical. Hence, the behavior of the Finnish labor share in the medium run is quite similar to the US counterpart (compare tables C.7 and 2); it even exhibits stronger pro-cyclicality.

Figure C.1: Labor share in Finland



The **red**, **blue** and **black** lines represent the raw series, the medium-to-long term component and the long-run trend, respectively. The data on the Finish labor share are taken from Jalava et al. (2006).

Figure C.2: Labor share in the UK

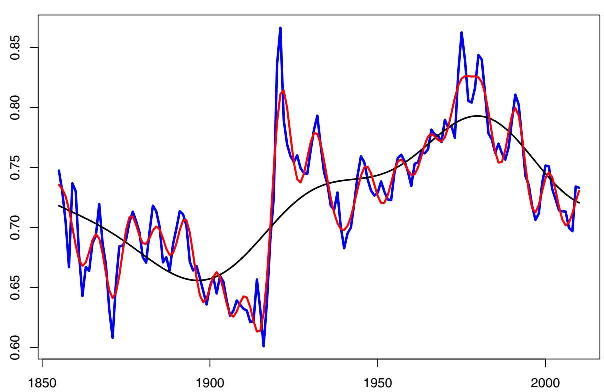
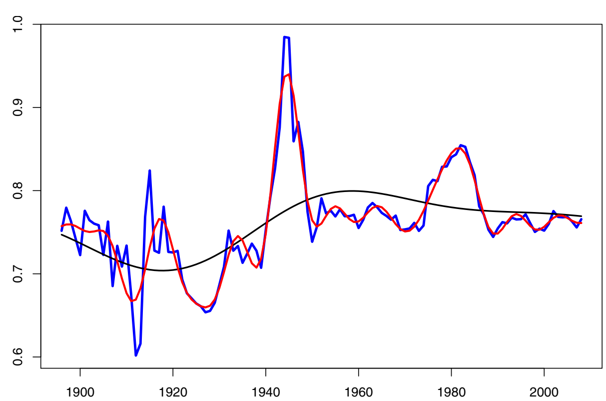
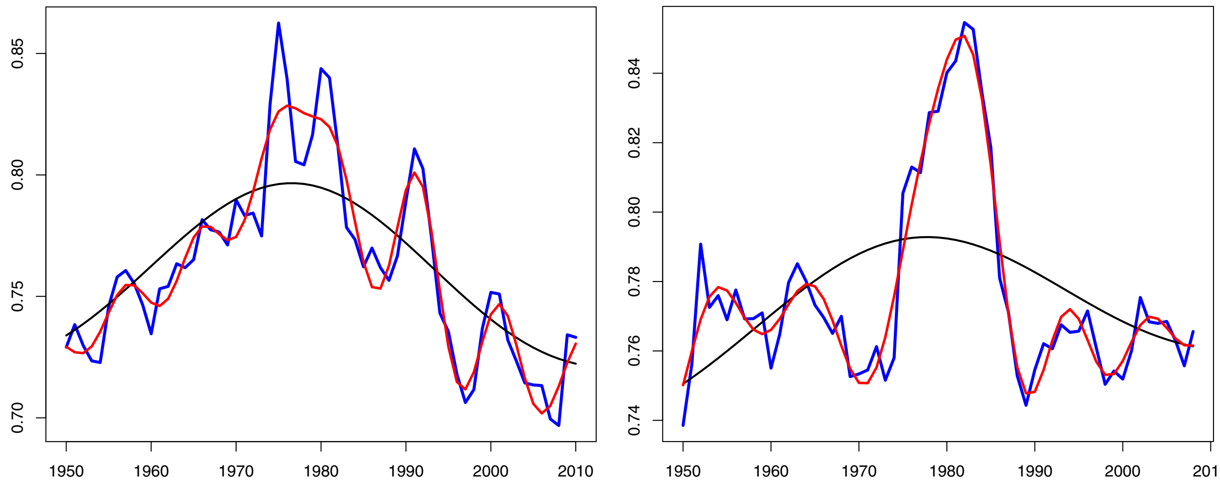


Figure C.3: Labor share in France



Note: The **red**, **blue** and **black** lines represent the raw series, the medium-to-long term component and the long-run trend, respectively. The data on the British and French labor share are taken from Piketty (2014).

Figure C.4: Labor share in the UK (left panel) and France (right panel) from the 1950s



The **red**, **blue** and **black** lines represents for raw, medium-term component and long-run trend, respectively.

Table C.6: Share of specific frequencies in the observed variance (in %)

Periodicity (in years)	≥ 50	8-50	≤ 8
		Finland	
excluding the mean	**79.5**	16.5	4.0
excluding a linear trend	15.3	**72.1**	12.6
excluding a quadratic trend	12.9	**73.4**	12.7
		UK	
excluding the mean	**66.3**	25.9	7.9
excluding a linear trend	42.0	**45.1**	12.9
excluding a quadratic trend	41.5	**45.5**	13.0
		France	
excluding the mean	35.6	**49.9**	14.5
excluding a linear trend	16.9	**65.9**	17.2
excluding a quadratic trend	14.0	**68.2**	17.8

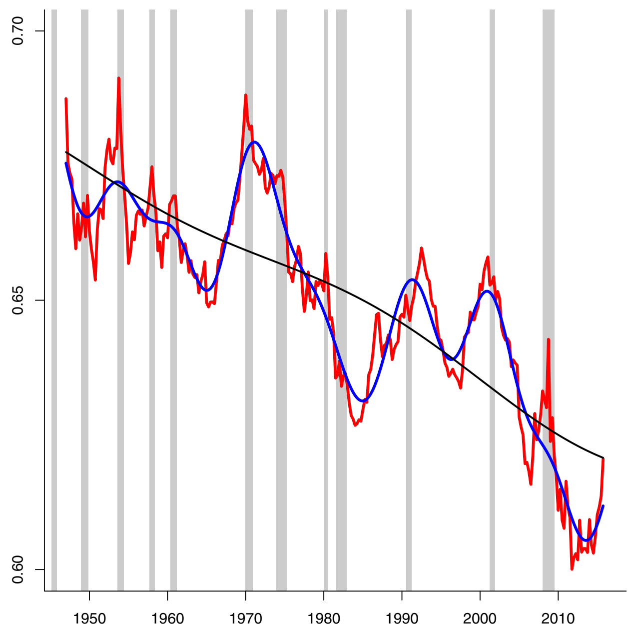
Note: the shares have been calculated using periodogram estimates. **Bold** indicates maximum value.

Table C.7: Features of Labor Share's Medium-Term Component in Finland

	σ_{LS_t}	$\sigma_{LS_t}/\sigma_{GDP_t}$	$\rho_{LS_t,LS_{t-1}}$	ρ_{LS_t,GDP_t}
Annual series	5.590	0.886	0.927	0.673
	(4.522,6.500)		(0.882,0.953)	(0.531,0.766)

D Additional Empirical Results

Figure D.1: The Quarterly Labor Share, Its Medium-Term Component and Long-Term Trend



Note: The **red**, **blue** and **black** lines represent the raw series, the medium-to-long term component and the long-run trend, respectively.

Table D.1: Characteristics of annual medium-term component of selected macroeconomic variables and cross-correlation with the labor share, $|\tau| \in [0, 10]$

	Correlation with the labor share					Medium-term characteristics			
		Max		*Min*					
	ρ_{LS_t,x_t}	$\rho_{LS_{t+\tau},x_t}$	τ	$\rho_{LS_{t+\tau},x_t}$	τ	σ_{x_t}	$\sigma_{x_t}/\sigma_{y_t}$	$\rho_{x_t,x_{t-1}}$	ρ_{y_t,x_t}
K_t/Y_t	-0.46	0.67	9	-0.83	-4	5.79	1.69	0.97	-0.55
	(-0.64, -0.26)	(0.52, 0.78)		(-0.9, -0.73)		(4.94, 6.52)		(0.95, 0.98)	(-0.7, -0.37)
C_t	0.66	0.68	-1	-0.51	-10	2.53	0.74	0.96	0.95
	(0.53, 0.77)	(0.54, 0.79)		(-0.63, -0.38)		(2.14, 2.86)		(0.93, 0.97)	(0.93, 0.96)
I_t	0.35	0.52	-2	-0.66	-10	6.44	1.88	0.92	0.8
	(0.15, 0.53)	(0.34, 0.66)		(-0.77, -0.5)		(5.45, 7.29)		(0.88, 0.95)	(0.74, 0.86)
C_t/Y_t	-0.38	0.54	-10	-0.66	-3	1.32	0.39	0.93	-0.78
	(-0.54, -0.19)	(0.33, 0.7)		(-0.78, -0.5)		(1.12, 1.51)		(0.9, 0.96)	(-0.85, -0.69)
I_t/Y_t	0.02	0.46	8	-0.57	-9	4.22	1.24	0.91	0.41
	(-0.21, 0.24)	(0.18, 0.68)		(-0.72, -0.36)		(3.5, 4.89)		(0.86, 0.95)	(0.23, 0.57)
RD_t/Y_t	-0.22	0.69	-9	-0.64	4	11.69	3.42	0.96	-0.08
	(-0.38, -0.04)	(0.53, 0.81)		(-0.74, -0.51)		(9.59, 13.55)		(0.93, 0.98)	(-0.3, 0.15)
RD_t^{NF}/Y_t	-0.02	0.59	-8	-0.67	5	6.4	1.87	0.94	-0.03
	(-0.23, 0.19)	(0.38, 0.74)		(-0.78, -0.51)		(5.37, 7.38)		(0.91, 0.96)	(-0.26, 0.21)
RD_t	-0.04	0.59	-8	-0.56	4	11.8	3.45	0.96	0.19
	(-0.24, 0.16)	(0.36, 0.74)		(-0.7, -0.4)		(9.94, 13.51)		(0.94, 0.98)	(-0.04, 0.41)
RD_t^{NF}	0.27	0.63	-3	-0.51	5	7.11	2.08	0.95	0.43
	(0.05, 0.47)	(0.47, 0.74)		(-0.67, -0.32)		(6, 8.16)		(0.92, 0.97)	(0.24, 0.6)
w_t^S/w_t^U	0.55	0.84	-3	-0.8	9	2.96	0.87	0.97	0.48
	(0.38, 0.71)	(0.74, 0.91)		(-0.93, -0.6)		(2.38, 3.49)		(0.94, 0.98)	(0.26, 0.67)
$Labor\ Prod_t$	0.34	0.5	3	-0.56	-10	3.05	0.89	0.97	0.64
	(0.16, 0.51)	(0.28, 0.67)		(-0.7, -0.37)		(2.77, 3.27)		(0.95, 0.98)	(0.48, 0.78)
L_t	0.35	0.48	-2	-0.44	-8	2.88	0.84	0.92	0.47
	(0.17, 0.51)	(0.3, 0.64)		(-0.65, -0.18)		(2.36, 3.41)		(0.88, 0.95)	(0.33, 0.59)
$L_t \times h_t$	0.38	0.67	-3	-0.35	10	3.13	0.91	0.92	0.58
	(0.2, 0.54)	(0.56, 0.77)		(-0.56, -0.08)		(2.53, 3.7)		(0.87, 0.95)	(0.47, 0.67)
C_t/K_t	0.44	0.8	-4	-0.7	9	5.58	1.63	0.97	0.49
	(0.24, 0.62)	(0.7, 0.87)		(-0.8, -0.54)		(4.71, 6.34)		(0.95, 0.98)	(0.3, 0.65)

Note: ρ_{y_t,x_t} and ρ_{LS_t,x_t} denote the contemporaneous cross-correlation for series x_t with output and the labor share. $\rho_{LS_{t+\tau},x_t}$ reflects to the correlation of variable x_t with labor share lagged by k period. For the labor share the highest and the lowest cross-correlation with each series are reported. $\rho_{x_t,x_{t-1}}$ and σ_{x_t} denote the first-order autocorrelation and standard deviation from the long-run trend, respectively.

Table D.2: P-values from the F signal test – Annual series

Cycle length	Excluding		
(in years)	Mean	Linear trend	Quadratic trend
87.00	[0.000]	[0.000]	[0.293]
43.50	[0.015]	[0.014]	[0.137]
29.00	[0.000]	[0.000]	[0.000]
21.75	[0.412]	[0.782]	[0.441]
17.40	[0.263]	[0.193]	[0.136]
14.50	[0.214]	[0.395]	[0.196]
12.43	[0.396]	[0.513]	[0.415]
10.88	[0.335]	[0.056]	[0.013]
9.67	[0.366]	[0.148]	[0.061]
8.70	[0.828]	[0.447]	[0.272]
7.91	[0.294]	[0.206]	[0.112]
7.25	[0.693]	[0.808]	[0.703]
6.69	[0.635]	[0.326]	[0.184]
6.21	[0.734]	[0.418]	[0.266]
5.80	[0.736]	[0.379]	[0.218]
5.44	[0.877]	[0.595]	[0.452]
5.12	[0.940]	[0.713]	[0.593]
4.83	[0.996]	[0.902]	[0.851]
4.58	[0.979]	[0.996]	[0.992]
4.35	[0.962]	[0.910]	[0.870]
4.14	[0.918]	[0.841]	[0.759]
3.95	[0.967]	[0.989]	[0.980]
3.78	[0.999]	[0.942]	[0.911]
3.63	[0.900]	[0.694]	[0.566]
3.48	[0.913]	[0.759]	[0.650]
3.35	[0.972]	[0.980]	[0.972]
3.22	[0.984]	[0.888]	[0.832]
3.11	[0.900]	[0.816]	[0.733]
3.00	[0.987]	[0.904]	[0.856]
2.90	[0.985]	[0.921]	[0.880]
2.81	[0.997]	[0.951]	[0.924]
2.72	[0.999]	[0.962]	[0.941]
2.64	[0.996]	[0.978]	[0.967]
2.56	[0.988]	[0.944]	[0.914]
2.49	[0.995]	[0.955]	[0.931]
2.42	[0.997]	[0.975]	[0.961]
2.35	[0.996]	[0.949]	[0.922]
2.29	[0.984]	[0.938]	[0.906]
2.23	[0.992]	[0.999]	[0.998]
2.18	[0.998]	[0.957]	[0.934]
2.12	[0.981]	[0.902]	[0.853]
2.07	[0.988]	[0.954]	[0.929]
2.02	[0.972]	[0.955]	[0.931]

Table D.3: P-values from the F signal test – Quarterly series

Cycle length	Excluding		
(in years)	Mean	Linear trend	Quadratic trend
69.00	[0.000]	[0.000]	[0.112]
34.50	[0.000]	[0.000]	[0.000]
23.00	[0.000]	[0.000]	[0.000]
17.25	[0.000]	[0.000]	[0.000]
13.80	[0.037]	[0.073]	[0.025]
11.50	[0.280]	[0.044]	[0.017]
9.86	[0.005]	[0.000]	[0.000]
8.63	[0.371]	[0.919]	[0.866]
7.67	[0.020]	[0.001]	[0.000]
6.90	[0.213]	[0.001]	[0.000]
6.27	[0.479]	[0.794]	[0.741]
5.75	[0.302]	[0.018]	[0.008]
5.31	[0.507]	[0.830]	[0.806]
4.93	[0.538]	[0.802]	[0.782]
4.60	[0.352]	[0.446]	[0.394]
4.31	[0.238]	[0.164]	[0.127]
4.06	[0.868]	[0.989]	[0.988]
3.83	[0.823]	[0.758]	[0.715]
3.63	[0.855]	[0.396]	[0.336]
3.45	[0.864]	[0.326]	[0.268]
3.29	[0.524]	[0.440]	[0.380]
3.14	[0.766]	[0.573]	[0.532]
3.00	[0.737]	[0.809]	[0.778]
2.88	[0.871]	[0.840]	[0.812]
2.76	[0.797]	[0.810]	[0.778]
2.65	[0.826]	[0.938]	[0.928]
2.56	[0.798]	[0.662]	[0.615]
2.46	[0.817]	[0.887]	[0.869]
2.38	[0.906]	[0.730]	[0.691]
2.30	[0.677]	[0.404]	[0.346]
2.23	[0.626]	[0.516]	[0.462]
2.16	[0.872]	[0.924]	[0.911]
2.09	[0.849]	[0.886]	[0.871]
2.03	[0.743]	[0.420]	[0.368]

E Additional Quantitative Results

We shall now provide a few additional quantitative results based on the model considered in the main text.

E.1 Impact of Parameter Variation on Labor Share at the BGP

Figure E.1 presents the impact of varying selected model parameters, holding other ones constant, on the BGP level of the labor share. All panels can be interpreted through the lens of equations (A.1) and (A.2):

$$\frac{\pi}{\pi_0} = \left(\frac{\lambda_b k}{\lambda_{b0} k_0}\right)^{\xi} \left(\frac{y}{y_0}\right)^{-\xi} \quad \Rightarrow \quad \hat{\pi} = \xi(\hat{\lambda}_b + \hat{k} - \hat{y}), \tag{A.1}$$

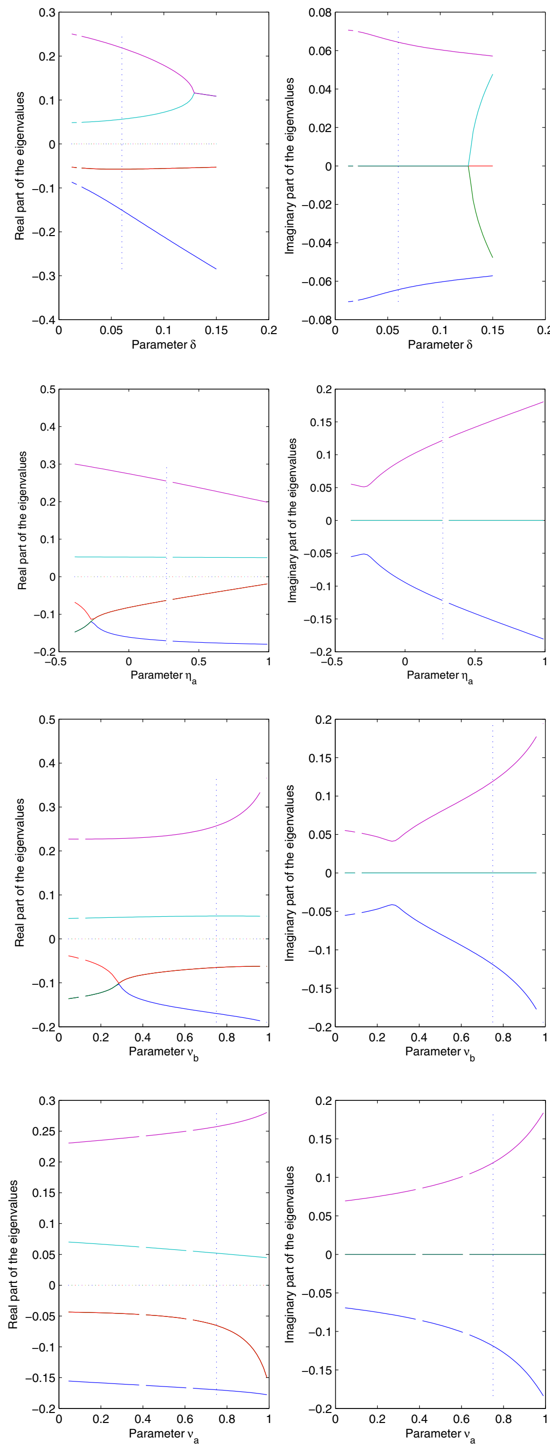
$$\frac{\pi}{1-\pi} = \frac{\pi_0}{1-\pi_0} \left(\frac{x}{x_0} \frac{\ell_{Y0}}{\ell_Y}\right)^{\xi} \quad \Rightarrow \quad \hat{\pi} = \xi(1-\pi)(\hat{x} - \hat{\ell}_Y). \tag{A.2}$$

As agents become less patient (higher ρ), R&D intensity falls, as does the labor share. Similar reasoning pertains to the inverse elasticity of substitution γ. That the result $\frac{\partial(1-\pi)}{\partial \eta_b} > 0 \mid \sigma < 1$ arises from the usual property that, under gross complements, improvements in capital augmenting technical change are labor biased; analogously $\frac{\partial(1-\pi)}{\partial \eta_a} < 0 \mid \sigma < 1$. Likewise, we have under gross complements: $\frac{\partial(1-\pi)}{\partial \nu_a} > 0$, $\frac{\partial(1-\pi)}{\partial \nu_b} < 0$. If capital depreciates faster, the capital (labor) share rises (falls).

Note that the lack of dependence of the BGP on ξ in the decentralized allocation follows from CES normalization (Klump and de La Grandville, 2000), coupled with the fact that we have calibrated the normalization constants to the BGP of the decentralized allocation.[35] This choice of point of normalization allows us to perfectly isolate changes in model dynamics (eigenvalues of the linearized system) due to changes in the elasticity of substitution from changes in the steady state location.

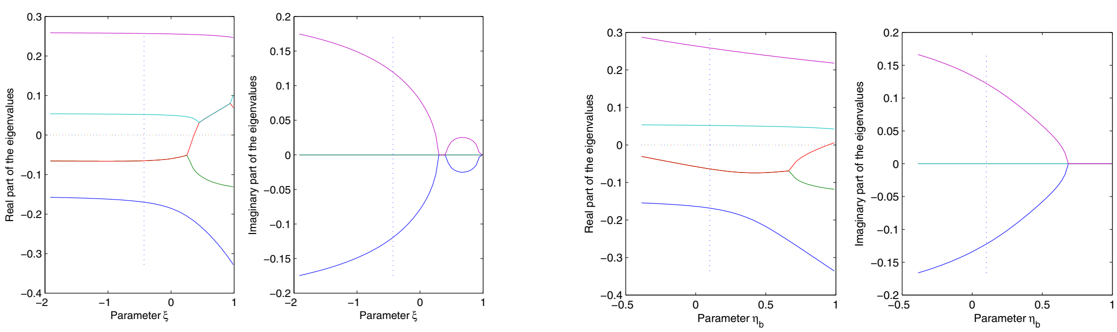
[35] For the decentralized steady state of the calibrated model to be completely insensitive to changes in the elasticity of substitution, one requires two assumptions. First, the CES production function is normalized. Second, it is normalized exactly at the steady state (so that $x^* = x_0$, $\ell_Y^* = \ell_{Y0}$, etc.) We do exactly that, thanks to which we isolate changes in model dynamics (eigenvalues) from changes in the steady state location, in a very clean way. It can be related to the findings of Klump and de La Grandville (2000) who isolated changes in the curvature of the production function from changes in unit factor productivity. For other choices of normalization constants, the BGP of the calibrated model would be sensitive to changes in the elasticity of substitution.

Figure E.1: Additional Bifurcation Figures



Note: The vertical dotted line in each graph represents the baseline calibrated parameter value.

Figure E.2: “Node–focus” Bifurcations



Note: The vertical dotted line in each graph represents the baseline calibrated parameter value.

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