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Endogenous labor share cycles: Theory and evidence[☆]

Jakub Growiec[a,b], Peter McAdam[c,*], Jakub Mućk[a,b]

[a] SGH Warsaw School of Economics, Poland
[b] Narodowy Bank Polski, Poland
[c] European Central Bank, Germany

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ABSTRACT

Based on long US time series we document a range of empirical properties of the labor's share of national income. We identify its substantial medium-to-long run, pro-cylical swings and show that most of its variance lies beyond business-cycle frequencies. We explore the extent to which these empirical regularities can be explained by a calibrated micro-founded, nonlinear growth model with normalized CES technology and endogenous labor- and capital augmenting technical change driven by purposeful directed R&D investments. We demonstrate that dynamic macroeconomic trade-offs created by arrivals of both types of new technologies can lead to prolonged swings in the labor share (and other model variables) due to oscillatory convergence to the balanced growth path as well as emergence of limit cycles via Hopf bifurcations. Both predictions are consistent with the empirical evidence.

1. Introduction

The proposition that labor's share of national income is stable has endured a long and chequered history. The classical economists – Smith, Ricardo, Marx – regarded labor shares as inherently variable, even in the long run. In stark contrast, the empirical observations of Cobb and Douglas (1928), Bowley (1937), Johnson (1954), Kaldor (1961) and others established the wide-spread constancy of such shares. This 'stylized' fact of stability lead to a benign neglect of the issue, only recently overturned given mounting evidence to the contrary. Our contribution to this literature is two-fold.

First, we try to establish (or re-establish) the stylized facts of labor share developments. We do so exploiting a frequency domain analysis. A common view is that labor share volatility is driven by business cycles (Hansen and Prescott, 2005), in a counter-cyclical manner, and subject in recent decades to a secular downward trend. Our analysis however demonstrates that business cycle fluctuations account for around 20% of the variance decomposition of the labor share. Of far greater importance are 'medium' and 'long run' frequencies. Another distinguishing fact is that whilst the high frequency component

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* Corresponding author.
E-mail addresses: peter.mcadam@ecb.europa.eu, peter.mcadam@ecb.int (P. McAdam).

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of the labor share is *counter-cyclical*, the (dominant) medium-run component is strongly *pro-cyclical*. Tests also show that the medium run labor share is highly persistent relative to the short-run component.

This takes us to our second contribution. Confronting these empirical features, we assess the extent to which a micro-founded endogenous growth model can account for them. The model is a generalization of Acemoglu (2003) with two R&D sectors giving rise to factor-augmenting innovations augmenting the "technology menu". We treat our model as a laboratory to assess mechanisms able to explain labor-share swings over the medium and long run.[1] Calibrating the nonlinear model on US data, we demonstrate that the interplay between endogenous arrivals of capital and labor-augmenting technologies leads to oscillatory convergence to the long-run growth path, and sometimes even to stable (self-sustaining endogenous) limit cycles.[2] The latter possibility (i.e., that the labor share oscillates indefinitely around a stable value) may be considered attractive in so far as it offers us a general theory of continuous labor-share movements as distinct from explanations associated to particular phenomena (e.g., reduced labor power, 'globalization', etc.)

Accordingly, our objective is to bridge the knowledge gap between what we observe of factor share movements and how we might model the mechanisms responsible for those movements. Indeed, the recent decline in labor income share poses a conspicuous challenge to theory. The usual macroeconomic paradigm of Cobb–Douglas production (unit elasticity of substitution, neutral technical change) coupled with isoelastic demand (leading to constant markups) leaves no room for the prolonged swings in factor shares observed in the data. Business-cycle models with variable markups or a cocktail of stochastic processes generate shares that stabilize rapidly around a constant mean (missing the high persistence and dominant low-frequency movements). Models endowed with a more general production specification (e.g., the neoclassical growth model with CES technology), on the other hand, do indicate a few critical tradeoffs; however, arguably the profession has not moved much beyond that.

The literature therefore tries to explain this phenomenon as departures from these benchmarks. For instance, through the impact of non-neutral technical change (Acemoglu, 2003; Autor et al., 2003; Jones, 2005b; Klump et al., 2007), structural transformation within the economy (Kongsamut et al., 2001; McAdam and Willman, 2013; Ngai and Pissarides, 2007; de Serres et al., 2002), shifting rents and shocks (Blanchard and Giavazzi, 2003; Blanchard, 1997) etc. Other explanations include the rise of offshoring of labor-intensive tasks (Elsby et al., 2013); increasing female labor force participation (Buera and Kaboski, 2012); changing patterns of firm size and age (Kyyrä and Maliranta, 2008); declines in relative prices for investment goods (Karabarbounis and Neiman, 2014); the tendency for capital returns to exceed economic growth rates (Piketty, 2014) and so on.[3] These explanations all have pros and cons of one sort or another (e.g., some explanations are tied to the value of production substitution parameters). Most of them however relate to technological changes, or can be viewed through the lens of technical developments (Boggio et al., 2010).

Against this literature, we find that a micro-founded endogenous growth model à la Acemoglu (2003) – with two R&D sectors giving rise to factor-augmenting innovations – is capable of supporting a self-sustaining cycle (a limit cycle) of low-frequency oscillations. We discuss the properties of that cycle and relate them to our earlier empirical findings and the growth literature more generally. For the baseline calibration of the model that we use, these oscillations are dampened ones. However, mild perturbations of that baseline can readily produce a self-sustaining cycle; e.g., if agents are sufficiently patient (a low discount rate) and/or flexible in allocating consumption across time (a high elasticity of intertemporal substitution), the subsequent arrivals of both types of innovations can generate limit cycle behavior. The cycle reflects the tension between the two R&D sectors (acceleration in each of them has conflicting impacts on the labor share). In such a case, the economy may converge to a stable cyclical path where all trendless macroeconomic variables (such as the labor share or the consumption–output ratio) oscillate indefinitely around the steady state. Such oscillations have a predetermined frequency and amplitude. Thus the model provides a plausible mechanism which is able to reproduce the observed labor share variations, including the ongoing long downward swing. At the same time, it predicts that this swing will eventually abate and reverse.

The paper proceeds as follows. Section 2 discusses some empirical properties of the US labor share. We find that the labor income share is highly persistent with a frequency decomposition skewed to the medium and long run. Section 3 discusses the model. This is an endogenous growth model with two R&D sectors giving rise to capital as well as labor augmenting innovations augmenting the "technology menu". Section 4 calibrates the model to US data and solves the nonlinear model. Next, we consider the dynamic properties of the model around the balanced growth path (BGP) in terms of oscillatory dynamics and the possible emergence of cycles, and uncovering the key channels involved. Section 6 concludes.

[1] Specifically, as far as we know, even though there exists a suite of endogenous growth models allowing for non-neutral technical change, their implications for medium-to-long run swings in the labor share have not yet been analyzed. Although, more generally, that economic activity may be subject to long waves of activity has proved influential following the seminal works of Kondratieff and Schumpeter.

[2] Articles, applications and surveys in this vein include Kaldor (1940), Goodwin (1951), Ryder and Heal (1973), Benhabib and Nishimura (1979), Dockner (1985), Feichtinger (1992), Benhabib and Perli (1994) and Ben-Gad (2003).

[3] Some authors have also envisaged an interesting approach whereby the labor share is described as a state variable in a model with Cobb–Douglas technology, which could be changed via purposeful spending on R&D, leading to 'factor-eliminating' technical change (Peretto and Seater, 2013; Zuleta, 2008).

2. Empirical evidence for medium-to-long swings in the labor share

We now explore some empirical properties of the US labor share.[4] We are of course not the first to do so (i.e., key contributions include Blanchard (1997) for Europe; Sturgill (2012), Elsby et al. (2013) and Oberfield and Raval (2014) for the US; Karabarbounis and Neiman (2014) globally). Our treatment however is notable in four respects.

1. Many such studies have concentrated on recent decades and, accordingly, have tended to emphasize the decline since the 1970s. In contrast we examine the broad historical evolution.[5]
2. We highlight the frequency decomposition of the labor share and, in so doing, the extent and importance of its medium-to-long run swings (or cycles). Many other contributions, by contrast, have not only concentrated on high-frequency (business-cycle) movements but have also used simple linear (or broken linear) trends to scrutinize features of the factor share. However, since Nelson and Plosser (1982) we know that assumption that some variables are stationary around a deterministic trend is too restrictive.
3. We derive some stylized facts about the labor share (variance, auto and cross-correlations) but, crucially, we do so again across the frequency domain.
4. We use our findings to motivate an endogenous growth model, which can account for and rationalize persistent fluctuations in the labor share.

Our main findings are the following. Most of the labor share's variance decomposition occurs not at business-cycle frequencies but at 'medium' and 'long-run' frequencies (≈ 80%). Stationarity and fractional integration tests corroborate this, showing that the labor share is highly persistent (indicative of slow mean reversion). Moreover, whilst the high frequency component of the labor share is *counter-cyclical*, the (dominant) medium-run component is strongly *pro-cyclical*.

2.1. The historical time series of the US labor share

The annual US labor income share is presented in Fig. 1.[6] Regarding data construction, we follow Gollin (2002) by adjusting the payroll share by proprietors' income (see Appendix A.1.1). The constructed series has all the properties usually identified in the literature: (i) it appears counter-cyclical,[7] and (ii) it has declined in recent decades.

However, using this long time series helps us appreciate that these two aspects are only *part* of the story: before the labor share began this decline, it showed something of an upward tendency. When examined over the entire period, the historical series arguably looks part of a long cycle. Thus, although the current labor share is back to values near those in 1929, it seems indeed reasonable to expect some eventual mean reversion (as appears to be the case at the end of the sample).

Moreover, according to Kaldor's (1961) stylized facts, factor shares should be stable over 'long' time periods, even if they fluctuate over the business cycle. However whilst the labor-share autoregressive parameter is largely close to but (statistically) below unity (see Table B.2), formal stationarity tests are inconclusive (see Table B.4). This ambiguity, this borderline stationarity, may reflect sample-size considerations, low test power in the presence of structural breaks or shifts, etc.

Notwithstanding, many time series exhibit too much long-range dependence to be strictly classified as $I(0)$, but are not $I(1)$ either. The ARFIMA model is designed to represent such series. If the fractional parameter[8] of a time series satisfies $d \in (0, 0.5)$, the auto-correlations decay more slowly than those of a stationary ARMA process (i.e., hyperbolically rather than geometrically). In short, the series is stationary but with long memory: shocks to the labor share (or its determinants) take a long time to decay.

This particular empirical finding (corroborated at the bottom of Table B.4) is consistent with our interpretation of factor income shares as ultimately mean reverting but driven by a long cycle, and (as demonstrated below) dominated by frequency movements beyond the business cycle.

2.2. Spectral analysis of labor income shares

Table 1 presents the estimated share of specific types of fluctuations in the total variance of the annual and quarterly series (employing three transformations). For the demeaned series, medium-frequency fluctuations are responsible for 46 – 50% of total volatility and the cycles mapped into the low-frequency pass are almost just as important (36 – 46%). Thus the total share of medium-to-long run frequencies is around 80%. As expected, de-trending the labor share series limits the contribution of low-frequency oscillations in the overall variance, and medium-term fluctuations become more important

[4] For compactness, some results in this section are relegated to appendices. Codes to replicate our empirical results are available.

[5] Indeed, there was a debate in the 1950s and 1960s on the explanations for the *increasing* labor share e.g., Solow (1958), Kravis (1959) and Ferguson and Moroney (1969).

[6] We additionally use quarterly series, available over 1947:1-2015:4, see Fig. D.1.

[7] See Table B.3.

[8] To illustrate fractional integration, consider $(1-L)^d X_t = \epsilon_t$ where L is the lag operator, $d \in \mathbb{R}$ is the differencing parameter and ϵ_t is the stationary short memory process. If $d = 1$, X_t is a random walk and integrated of order one, $I(1)$. If $d = 0$, X_t is white noise and weakly stationary, $I(0)$.

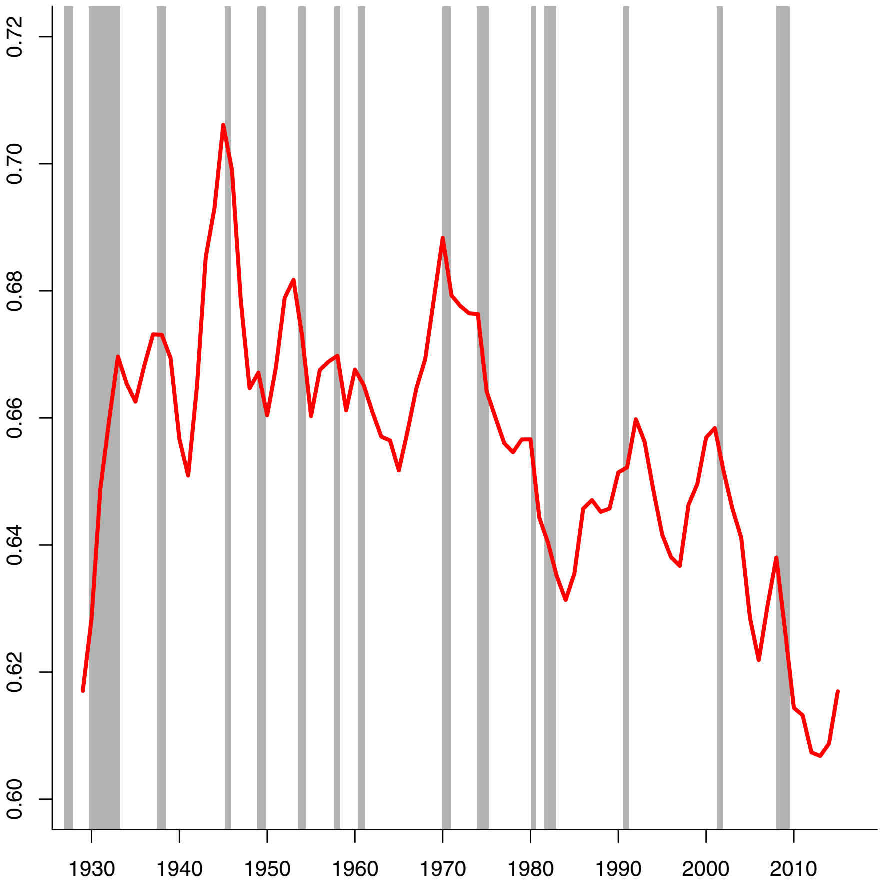

Fig. 1. The annual US labor share, 1929–2015. Note: shaded areas represent recessions according to the NBER chronology overlaid at quarterly frequency. Summary statistics for the annual and quarterly labor income share are given in Table B.1.

instead, with their share about 58% and 70% for the series de-trended by a linear and quadratic trend, respectively. Business-cycle fluctuations, by contrast account for only 8–29% of the total variance.[9]

Looking at the periodogram estimates for both annual and quarterly data (Fig. 2), we also note there are two dominant frequencies of fluctuations in the US labor share: (a) medium-term cycles lasting around 30 years, and (b) the long-run stochastic trend, whose length reaches beyond the 80 years mark. As opposed to business-cycle models, the mechanisms present in our model are able to generate swings of either of these frequencies.

2.3. Stylized facts: labor share's medium-term fluctuations

The medium and long-term component extracted from the labor share is depicted in Fig. 3.[10] The former component is responsible for a significant part of the overall volatility of the series and has an important contribution to the scale of deviation from long run trend at the turning points. Although isolating the medium- and high-frequency cycles reduces the volatility substantially, the remaining smoothed long-run trend is still hump-shaped (with a peak around the late 1950s/early 1960s).

Table 2 reports the main features of the medium and short-term component of the labor share using moments of the filtered series. Whilst volatility is similar, a stark difference is the counter-cyclical (or a-cyclical) short-run labor share, viewed against the strongly pro-cyclical medium-term component (at 0.6 or above). Indeed, many macroeconomic variables underpinning the labor share (e.g., investment, consumption, hours worked, employment) are virtually all pro-cyclical in the medium term (see Table D.1).

[9] In Tables D.2 and D.3 we test the significance of the spectral density peaks following Wei (2006). The null (alternative) hypothesis is that at given frequency there is noise (a significant cycle). The tables confirm that medium-term fluctuations are very important, relative to other ranges.

[10] The choice of method for extracting the medium-term component from the data is mostly determined by the frequency domain in question. Following earlier work on medium-term cycles (e.g., Comin and Gertler, 2006), we apply the Christiano and Fitzgerald (2003) (CF) approximation of the ideal band-pass filter. The general strategy of isolating the medium-term component is the following. We transform our data into log differences and then apply the band-pass (CF) filter. Next, we cumulate the filtered data and demean. This increases filter efficiency as compared to applying the filtering procedure directly to log levels. The extracted series represent percentage deviations from the long-run trend.

Table 1
Share of specific frequencies in total variance of the labor share (in %).

	Annual			Quarterly		
Periodicity (in years)	≥ 50	8–50	≤ 8	≥ 50	8–50	≤ 8
Excluding the mean	**46.2**	45.5	8.4	36.1	**49.8**	14.2
Excluding a linear trend	23.7	**58.4**	17.9	8.7	**66.6**	24.7
Excluding a quadratic trend	2.9	**69.3**	27.8	1.6	**69.2**	29.2

Note: the shares have been calculated using periodogram estimates. **Bold** indicates maximum value. Following Comin and Gertler (2006) we define high (periodicity below 8 years), medium (periodicity between 8 and 50 years) and low-frequency oscillations (periodicity above 50 years).

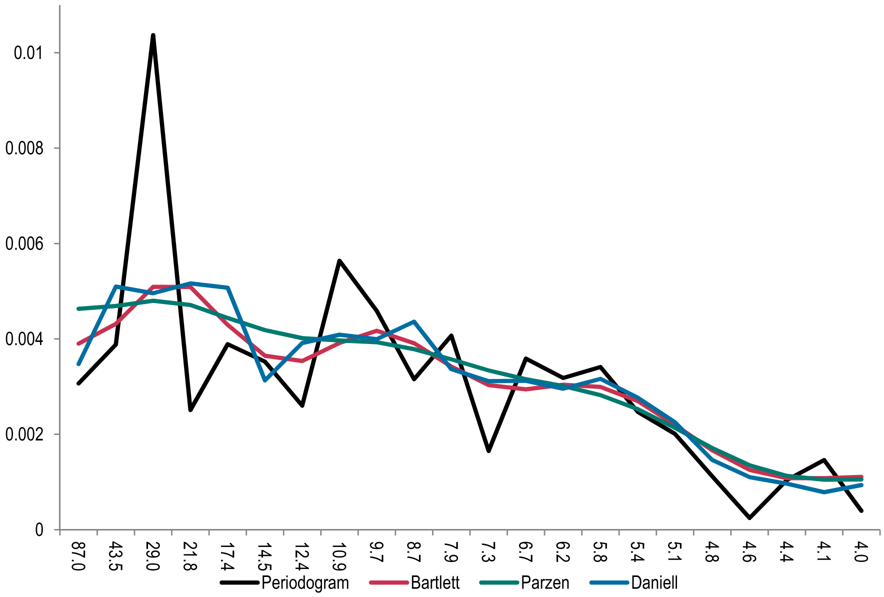

Fig. 2. Periodogram estimates: annual series excluding a quadratic trend.

Finally, note that the short-run component (i.e., that derived from first differencing or standard filtering) has relatively weak persistence (0.3–0.7) in comparison to the medium run component which has a persistence parameter above 0.9. This value is relatively similar to the persistence of the raw series, which is not so surprising given our earlier conclusion that the variance decomposition of the series is dominated by its medium run frequencies.

2.4. Summary

We demonstrated that movements in the labor share are dominated by medium and low frequencies. Furthermore, such movements are distinct from business-cycle fluctuations since they (and their determinants) are pro-cyclical and characterized by slow, cyclical mean reversion. These 'facts' matter for our modelling choices: to understand the fundamental dynamic underpinning labor shares, requires a framework and method that can speak to those features. Moreover, outside of the business cycle, we typically consider activity as being endogenously driven by technical progress (inter alia, Gancia and Zilibotti, 2009). And it is to such a model that we now turn.

3. Model

The framework is a generalization of Acemoglu's (2003) model with capital and labor augmenting R&D which, in turn, draws on the earlier induced innovation literature from Kennedy (1964) onwards, as well as Romer (1990) and Jones (1999), the Dixit–Stiglitz monopolistic competition framework and so on. The distinct features of our treatment, though, are worth pointing out and are the following:

(i) our model is non-scale: both R&D functions are specified in terms of percentages of population employed in either R&D sector (as opposed to Acemoglu where the R&D functions are specified in terms of total R&D employment);

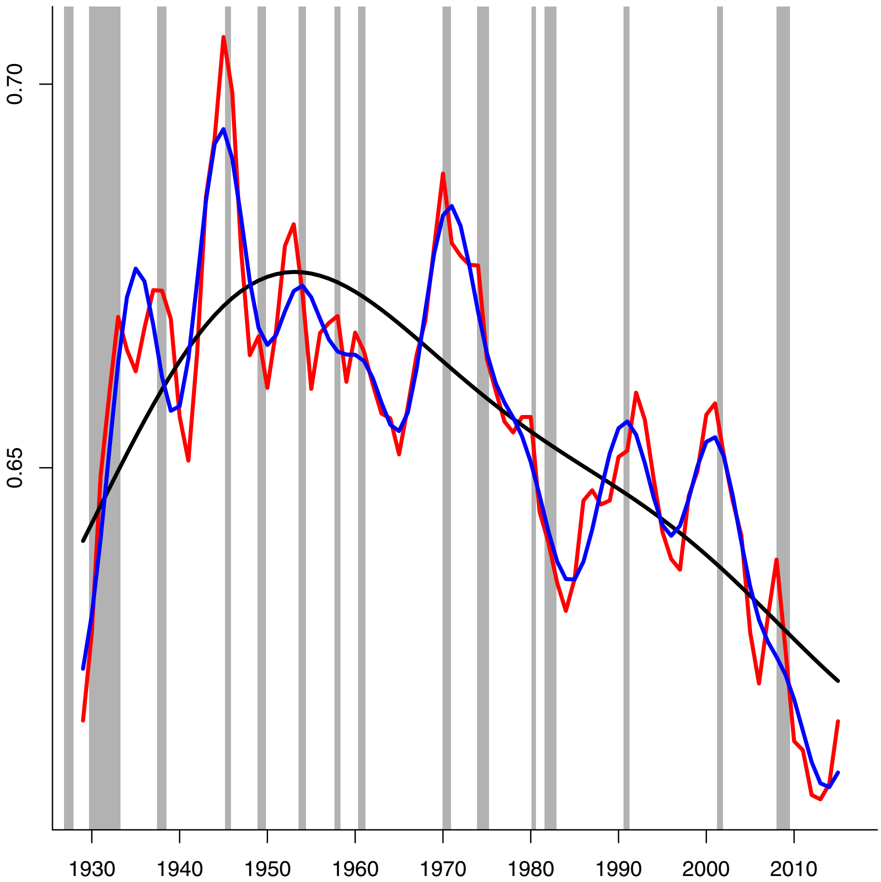

Fig. 3. Medium-term component & long-term stochastic trend of annual US labor share. Note: the red, blue and black lines represent the raw series, the medium-to-long term component and the long-run trend, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

(ii) we also assume R&D workers are drawn from the same pool as production workers[11];

(iii) we assume more general R&D technologies which allow for mutual spillovers between both R&D sectors (cf. Li, 2000) and for concavity in capital-augmenting technical change;

(iv) in contrast to Acemoglu (2003), the BGP growth rate g in our model depends on preferences via ℓ_Y (labor in aggregate production). The tradeoff is due to drawing researchers from the same employment pool as production workers (a tradeoff not present in his model) and;

(v) we use *normalized* CES production functions.[12]

These changes make our setup less restrictive as regards developments in factors and factor prices and, thus, make the identification of cycles more plausible.[13] In our case with gross complementarity between capital and labor in the aggregate CES production function, oscillations in factor shares – and other model variables – appear endogenously as an outcome

[11] Acemoglu (2003) assumes that labor supply in the production sector is inelastic and R&D is carried out by a separate group of “scientists” who cannot engage in production labor. Our assumption affects the tension between both R&D sectors by providing R&D workers with a third option, the production sector.

[12] Normalization implies representing the production function and factor demands in consistent index number form. Without normalization, it can be shown that the production parameters have no economic interpretation since they are dependent on the normalization point and on the elasticity of substitution itself. This undermines estimation and comparative statics. See de La Grandville (1989) and Klump and de La Grandville (2000) for the seminal contributions, and León-Ledesma et al. (2010) for an econometric analysis. Bartelme and Gorodnichenko (2015) use the normalized function to examine the link between industry and aggregate productivity. See Cantore et al. (2014), Palivos and Karagiannis (2010), Irmen (2011) for general background on the importance of the substitution elasticity. Moreover, we confine ourselves to constant returns, consistent with much of the aggregate evidence, e.g., Basu and Fernald (1997).

[13] The literature on endogenous cycles in growth models (albeit usually conducted in systems of lower dimension than ours) has identified various discontinuities and ad-hoc mechanisms to generate cycles: e.g., non concavities, adjustment costs, delay functions, information asymmetries and stickiness, high discount rates, strong control-state interactions, etc. Our framework, being micro-founded and less restrictive along the dimensions indicated, makes the analysis and identification of cycles accordingly, we believe, more ‘plausible’.

Table 2
Features of labor share's medium and short-term components.

	σ_{LS_t}	$\sigma_{LS_t}/\sigma_{GDP_t}$	$\rho_{LS_t,LS_{t-1}}$	ρ_{LS_t,GDP_t}
Medium terms				
Annual series	1.518	0.444	0.939	0.634
	(1.285,1.722)		(0.909,0.962)	(0.483,0.759)
Quarterly series	1.547	0.449	0.996	0.583
	(1.445,1.652)		(0.995,0.997)	(0.504,0.654)
Short terms†				
Annual series	0.660	0.431	0.325	-0.098
	(0.538, 0.767)		(0.142, 0.502)	(-0.316, 0.116)
quarterly series	0.785	0.513	0.736	-0.185
	(0.700, 0.869)		(0.673, 0.795)	(-0.289, -0.076)

Note: σ_{LS_t} and $\sigma_{LS_t}/\sigma_{GDP_t}$ denotes volatility in absolute term (percentage deviation from the long-run trend) and relative term (as a ratio to the GDP's volatility). $\rho_{LS_t,LS_{t-1}}$ and ρ_{LS_t,GDP_t} stand for the first-order autocorrelation and contemporaneous co-movement, respectively. 95% bootstrapped confidence intervals (based on 5000 replications) in parentheses. †We use three measures of the short run component: first-differencing, the CF filter (all fluctuations with periodicity between 2 and 8 years) and, in this table, the HP filter ($\lambda = 1600$ and $\lambda = 6.25$ for quarterly and annual series, respectively). Results for the other filtered short-run series are given in Table B.5.

of the interplay between labor and capital augmenting R&D. The cycle reflects the tension between the two R&D sectors (acceleration in each of them has conflicting impacts on the labor share, Irmen and Tabaković, 2017).

An additional departure relative to the existing literature is that we provide a full and rigorous calibration of the model based on US data. Among other dynamic properties, we can therefore assess model fit in terms of the lengths of cycles.

3.1. Aggregate production

Assume aggregate production is of the normalized CES form:[14]

$$Y = Y_0\left(\pi_0\left(\frac{\lambda_b}{\lambda_{b0}}\frac{K}{K_0}\right)^{\xi} + (1-\pi_0)\left(\frac{\lambda_a}{\lambda_{a0}}\frac{L_Y}{L_{Y0}}\right)^{\xi}\right)^{\frac{1}{\xi}} \tag{1}$$

where $\sigma = (1-\xi)^{-1} \in (0,\infty)$ is the elasticity of factor substitution. Under normalization, benchmark values are assigned to output, capital and labor (Y_0, K_0, L_0)[15] and technology. Terms λ_a and λ_b represent the maximum degree of factor augmentation along the "technology menu" and grow as an outcome of factor augmenting R&D. Henceforth without loss of generality we set $\lambda_{b0} = 1$. Under perfect competition the capital share is determined by the margins of capital augmenting technology and the capital-output ratio, whose qualitative impact is determined by sign$\{\xi\}$:

$$\pi = \pi_0\left(\frac{\lambda_b K}{\lambda_{b0}K_0}\right)^{\xi}\left(\frac{Y}{Y_0}\right)^{-\xi}.$$

With constant returns, the labor share equals $1-\pi$.

3.2. R&D

We assume that new, factor augmenting innovations are created endogenously by the respective R&D sectors augmenting the technology menu by increasing the underlying parameters λ_a, λ_b:

$$\dot{\lambda}_a = A\left(\lambda_a\lambda_b^{\phi}x^{\eta_a}\ell_a^{\nu_a}\right), \tag{2}$$

$$\dot{\lambda}_b = B\left(\lambda_b^{1-\omega}x^{\eta_b}\ell_b^{\nu_b}\right) - d\lambda_b, \tag{3}$$

[14] In terms of the distinction between 'local' and 'global' production functions (Jones, 2005b), an aggregate CES production function can be justified if new production techniques are independently drawn from identical Weibull distributions, Growiec (2013). See our working paper version of this article (Growiec et al., 2015) for a detailed derivation.

[15] Although some have also considered human capital accumulation in constructing and analyzing the labor share (key references being Krueger, 1999 and Zuleta, 2008), we abstract from human capital. This was largely done for simplicity since, amongst other things, the introduction of human capital as a separate production factor raises issues of using and discriminating among different hierarchies of multi-level production functions, see León-Ledesma et al. (2012). One, though, might consider the labor inputs to be human-capital adjusted (i.e., similar to how the KLEMS database defines 'labor services'). Likewise, one can consider the labor augmenting technical progress term as capturing some of the effects of human capital on the labor input.

where ℓ_a and ℓ_b are the shares (or "research intensity") of population employed in labor- and capital augmenting R&D, respectively, with $\ell_a + \ell_b + \ell_Y = 1$, and $\ell_Y L = L_Y$, etc. Term $x \equiv k(\lambda_b/\lambda_a)$ is the effective capital-labor ratio where $k = K/L$.[16] The terms ℓ_a, ℓ_b, ℓ_Y and x are constant along the BGP. The long-term endogenous growth engine is located in the linear labor augmenting R&D equation. To fulfill the requirement of the existence of a BGP along which the growth rates of λ_a and λ_b are constant, we assume $\eta_b\phi + \eta_a\omega \neq 0$.[17]

Parameters A and B capture the unit productivity of the labor- and capital augmenting R&D process, respectively. Parameter ϕ captures the spillover from capital to labor augmenting R&D.[18] Parameter ω measures the degree of decreasing returns to scale in capital augmenting R&D. By assuming $\omega \in (0, 1)$ we allow for the "standing on shoulders" effect in capital augmenting R&D, albeit we limit its scope insofar as it is less than proportional to the existing technology stock (Jones, 1995).

We assume capital augmenting developments are subject to gradual decay at a rate $d > 0$, which mirrors susceptibility to obsolescence and embodied character of capital augmenting technologies, Solow (1960). This assumption is critical for the asymptotic constancy of unit capital productivity λ_b, and thus for the existence of a BGP with purely labor augmenting technical change.

3.2.1. Duplication externalities

A key insight of the endogenous growth literature is that R&D activity may be subject to duplication externalities (Jones, 1995; Stokey, 1995). This is captured by parameters ν_a, $\nu_b \in (0, 1]$: the higher is ν the lower the extent of duplication. This negative externality may arise from many sources, e.g., patent races and patent protection. A race to secure a lucrative (e.g., medical) patent, for instance, may imply large decentralized, overlapping scientific resources. Likewise, with stringent patent protection, wasteful duplication may arise since firms cannot directly build on patented technology (having first to reinvent/imitate it). On the other hand, with more patent protection, there could be less duplication because each research project gives the firm more leverage due to its patentability and the exclusion of competition. The net effect is unclear.

These externalities are important in our analysis. Indeed, we are not aware of any study which distinguishes between duplication externalities in labor- and capital augmenting R&D. This raises the question of whether $\nu_a = \nu_b$, though a defensible prior, makes sense. For instance, duplication externalities could be stronger in labor augmenting R&D, in so far as there is greater scope for patent protection when the technology is embodied in capital goods and subject to obsolescence. Accordingly, we explore several $\{\nu_a, \nu_b\}$ scenarios.

3.2.2. Nested forms in R&D accumulation

To put our forms in context, observe that switching off all externalities and spillovers in (2)–(3) by setting $d = \omega = \eta_a = \eta_b = 0$ and $\nu_a = \nu_b = 1$ retrieves the original specification of R&D in Acemoglu (2003).[19] Moreover, compared with models which use aggregate Cobb–Douglas, equation (3) is akin to Jones' (1995) formulation of the R&D sector, generalized by adding obsolescence and the capital-labor term. Thus, setting $d = \eta_b = 0$ retrieves Jones' original specification. And equation (2) is the same as in Romer (1990) but scale-free (i.e., it features a term in ℓ_b instead of $\ell_b \cdot L$) and with effective capital-labor in R&D and a direct spillover from λ_b; setting $\phi = \eta_a = 0$ retrieves the scale-free version of Romer (1990), cf. Jones (1999).

3.3. The decentralized allocation

The construction of the decentralized allocation of the model draws from Romer (1990), Acemoglu (2003), and Jones (2005a). In particular, we use the Dixit–Stiglitz monopolistic competition setup and the increasing variety framework of the R&D sector. The general equilibrium is obtained as an outcome of the interplay between: households; final goods producers; aggregators of bundles of capital and labor-intensive intermediate goods; monopolistically competitive producers of differentiated capital and labor-intensive intermediate goods; and competitive capital and labor augmenting R&D firms. We discuss these agents in turn in the following subsections.

3.3.1. Households

Assume the representative household maximizes discounted CRRA utility:

$$\max \int_0^\infty \frac{c^{1-\gamma} - 1}{1-\gamma} e^{-(\rho-n)t} dt \tag{4}$$

subject to the budget constraint:

$$\dot{v} = (r - \delta - n)v + w - c, \tag{5}$$

---

[16] Thus, observe, there are *mutual* spillovers between both R&D sectors, with no prior restriction on their strength: $\dot{\lambda}_a = A\lambda_a^{1-\eta_a}\lambda_b^{\phi+\eta_a}k^{\eta_a}\ell_a^{\nu_a}$ and $\dot{\lambda}_b = B\lambda_a^{-\eta_b}\lambda_b^{1-\omega+\eta_b}k^{\eta_b}\ell_b^{\nu_b} - d\lambda_b$.

[17] All our qualitative results also go through for the special case $\eta_a = \eta_b = 0$, which fully excludes the presence of the effective capital-labor term, x, in R&D. The current inequality condition is not required in such case.

[18] There are no a priori restrictions on sign$\{\phi\}$. In our baseline calibration we assume $\phi > 0$, indicating that more efficient use of physical capital in the economy also increases the productivity of labor augmenting R&D. See Li (2000) for a thorough discussion of the role of cross-sectoral spillovers in growth models with two R&D sectors.

[19] Furthermore, Acemoglu (2003) assumes scientists to be a separate input from labor, and considers an additional case where both types of innovations are subject to decay at a rate d.

where $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution, $\rho > 0$ is the rate of time preference, $n > 0$ is the (exogenous) growth rate of the labor supply, and $v = V/L$ is the household's per-capita holding of assets, $V = K + p_a\lambda_a + p_b\lambda_b$. The representative household is the owner of all capital and also holds the shares of monopolistic producers of differentiated capital and labor-intensive intermediate goods. Capital is rented at a net market rental rate equal to the gross rental rate less depreciation: $r - \delta$. Solving the household's optimization problem yields the consumption Euler equation:

$$\hat{c} = \frac{r - \delta - \rho}{\gamma}, \tag{6}$$

where $\hat{c} = \dot{c}/c = g$ (the per-capita growth rate).

3.3.2. Final goods producers

The role of final goods producers is to generate the output of final goods (which are then either consumed by the representative household or saved and invested, leading to physical capital accumulation), taking bundles of capital and labor-intensive intermediate goods as inputs. They operate in a perfectly competitive environment, where both bundles are remunerated at market rates p_K and p_L, respectively.

The final goods producers operate a normalized CES technology:

$$Y = Y_0\left(\pi_0\left(\frac{Y_K}{Y_{K0}}\right)^{\xi} + (1-\pi_0)\left(\frac{Y_L}{Y_{L0}}\right)^{\xi}\right)^{\frac{1}{\xi}}. \tag{7}$$

The first order condition implies that final goods producers' demand for capital and labor-intensive intermediate goods bundles satisfies,

$$p_K = \pi\frac{Y}{Y_K}, \quad p_L = (1-\pi)\frac{Y}{Y_L}, \tag{8}$$

where the share term $\pi = \pi_0(\frac{Y_K}{Y_{K0}}\frac{Y_0}{Y})^{\xi}$ is the elasticity of final output with respect to Y_K.

3.3.3. Aggregators of capital- and labor-intensive intermediate goods

There are two symmetric sectors in the economy, whose role is to aggregate the differentiated (capital or labor-intensive) goods into the bundles Y_K and Y_L demanded by final goods producers. It is assumed that the differentiated goods are imperfectly substitutable (albeit gross substitutes). The degree of substitutability is captured by parameter $\varepsilon \in (0, 1)$:

$$Y_K = \left(\int_0^{N_K} X_{Ki}^{\varepsilon} di\right)^{\frac{1}{\varepsilon}}. \tag{9}$$

Aggregators operate in a perfectly competitive environment and decide upon their demand for intermediate goods, the price of which will be set by the respective monopolistic producers (discussed below).

For capital-intensive bundles, the aggregators maximize

$$\max_{X_{Ki}}\left\{p_K\left(\int_0^{N_K} X_{Ki}^{\varepsilon} di\right)^{\frac{1}{\varepsilon}} - \int_0^{N_K} p_{Ki}X_{Ki} di\right\}, \tag{10}$$

for a continuum of measure N_K of capital-intensive intermediate goods producers. Optimization implies the following demand curve:

$$X_{Ki} = x_K(p_{Ki}) = \left(\frac{p_{Ki}}{p_K}\right)^{\frac{1}{\varepsilon-1}} Y_K^{\frac{1}{\varepsilon}}. \tag{11}$$

Symmetrically, there is also a continuum of measure N_L of labor-intensive intermediate goods producers. The demand curve for their products satisfies,

$$X_{Li} = x_L(p_{Li}) = \left(\frac{p_{Li}}{p_L}\right)^{\frac{1}{\varepsilon-1}} Y_L^{\frac{1}{\varepsilon}}. \tag{12}$$

3.3.4. Producers of differentiated intermediate goods

It is assumed that each of the differentiated capital or labor-intensive intermediate goods producers, indexed by $i \in [0, N_K]$ or $i \in [0, N_L]$ respectively, has monopoly over its specific variety. It is therefore free to choose its preferred price p_{Ki} or p_{Li}. These firms operate a simple linear technology, employing either only capital or only labor.

For capital-intensive intermediate goods producers, the production function is $X_{Ki} = K_i$. Capital is rented at the gross rental rate r. The optimization problem is:

$$\max_{p_{Ki}}\,(p_{Ki}X_{Ki} - rK_i) = \max_{p_{Ki}}(p_{Ki} - r)x_K(p_{Ki}). \tag{13}$$

The optimal solution implies $p_{Ki} = r/\varepsilon \ \forall i \in [0, N_K]$. This implies symmetry across all differentiated goods: they are sold at equal prices, thus their supply is also identical, $X_{Ki} = \bar{X}_K \forall i$. Given this regularity, market clearing implies:

$$K = \int_0^{N_K} K_i di = \int_0^{N_K} X_{Ki} di = N_K \bar{X}_K \qquad Y_K = N_K^{\frac{1-\varepsilon}{\varepsilon}} K. \tag{14}$$

The demand curve implies that the price of intermediate goods is linked to the price of the capital-intensive bundle as in $p_K = p_{Ki} N_K^{\frac{\varepsilon-1}{\varepsilon}} = (r/\varepsilon) N_K^{\frac{\varepsilon-1}{\varepsilon}}$.

Symmetrically, in the labor-intensive sector, the production function is $X_{Li} = L_{Yi}$. Employees are remunerated at the market wage rate w. The total labor supply is given by $L_Y = \ell_Y L = \int_0^{N_L} L_{Yi} di$. Optimization yields $p_{Li} = w/\varepsilon$. By symmetry, we also obtain:

$$L_Y = \int_0^{N_L} X_{Li} di = N_L \bar{X}_L \qquad Y_L = N_L^{\frac{1-\varepsilon}{\varepsilon}} L_Y. \tag{15}$$

The respective prices satisfy $p_L = p_{Li} N_L^{\frac{\varepsilon-1}{\varepsilon}} = (w/\varepsilon) N_L^{\frac{\varepsilon-1}{\varepsilon}}$.

Finally, aggregating across all the intermediate goods producers, we obtain that their total profits are equal to $\Pi_K N_K = rK\left(\frac{1-\varepsilon}{\varepsilon}\right)$ and $\Pi_L N_L = wL_Y\left(\frac{1-\varepsilon}{\varepsilon}\right)$ for capital and labor-intensive goods respectively. Streams of profits per person in the representative household are thus $\pi_K = \Pi_K/L$ and $\pi_L = \Pi_L/L$, respectively. Hence, the total remuneration channeled to the capital-intensive sector equals $p_K Y_K = (r/\varepsilon)K = rK + \Pi_K N_K$, whereas the total remuneration channeled to the labor-intensive sector equals $p_L Y_L = (w/\varepsilon)L_Y = rL_Y + \Pi_L N_L$.

Comparing these results to the optimization problem of the final goods firms leads to,

$$r = \varepsilon \pi \frac{Y}{K} = \varepsilon \pi_0 \left(\frac{Y}{K}\right)^{1-\xi} \left(\frac{Y_0}{K_0}\right)^{\xi} \left(\frac{N_K}{N_{K0}}\right)^{\xi\left(\frac{1-\varepsilon}{\varepsilon}\right)}, \tag{16}$$

$$w = \varepsilon(1-\pi)\frac{Y}{L_Y} = \varepsilon(1-\pi_0)\left(\frac{Y}{L_Y}\right)^{1-\xi} \left(\frac{Y_0}{L_{Y0}}\right)^{\xi} \left(\frac{N_L}{N_{L0}}\right)^{\xi\left(\frac{1-\varepsilon}{\varepsilon}\right)}, \tag{17}$$

$$\frac{p_K}{p_L} = \frac{\pi}{1-\pi}\frac{Y_L}{Y_K} = \frac{\pi}{1-\pi}\frac{L_Y}{K}\left(\frac{N_L}{N_K}\right)^{\frac{1-\varepsilon}{\varepsilon}} = \frac{r}{w}\left(\frac{N_L}{N_K}\right)^{\frac{1-\varepsilon}{\varepsilon}}. \tag{18}$$

In equilibrium, factor shares then amount to,

$$\pi = \pi_0 \left(\frac{KY_0}{K_0 Y}\right)^{\xi} \left(\frac{N_K}{N_{K0}}\right)^{\xi\left(\frac{1-\varepsilon}{\varepsilon}\right)}, \tag{19}$$

$$1-\pi = (1-\pi_0)\left(\frac{L_Y Y_0}{L_{Y0} Y}\right)^{\xi} \left(\frac{N_L}{N_{L0}}\right)^{\xi\left(\frac{1-\varepsilon}{\varepsilon}\right)}. \tag{20}$$

Hence, the aggregate production function, obtained after incorporating all these choices into (7), and using the definitions $\lambda_b = N_K^{\frac{1-\varepsilon}{\varepsilon}}$ and $\lambda_a = N_L^{\frac{1-\varepsilon}{\varepsilon}}$, yields,

$$Y = Y_0\left(\pi_0\left(\frac{\lambda_b K}{\lambda_{b0} K_0}\right)^{\xi} + (1-\pi_0)\left(\frac{\lambda_a L_Y}{\lambda_{a0} L_{Y0}}\right)^{\xi}\right)^{\frac{1}{\xi}} \tag{1'}$$

which coincides with the aggregate production function (1).

3.3.5. Capital and labor augmenting R&D firms

The role of capital and labor augmenting R&D firms is to produce innovations which increase the variety of available differentiated intermediate goods (N_K or N_L), and thus indirectly also λ_b and λ_a. Patents never expire, and patent protection is perfect. R&D firms sell these patents to the representative household which sets up a monopoly for each new variety. Patent price, p_b or p_a, which reflects the discounted stream of future monopoly profits, is set at the competitive market. There is free entry to R&D.

R&D firms employ labor only: $L_a = \ell_a L$ and $L_b = \ell_b L$ workers are employed in the labor- and capital augmenting R&D sectors, respectively. There is also an externality from the total physical capital stock in the economy, working through the effective capital-labor ratio in the R&D production function. Furthermore, the R&D firms perceive their production technology as linear in labor, while in fact it is concave due to duplication externalities.

Incorporating these assumptions and recalling that $x = k(\lambda_b/\lambda_a)$, capital augmenting R&D firms maximize:

$$\max_{\ell_b} \left(p_b\dot{\lambda}_b - w\ell_b\right) = \max_{\ell_b}\left((p_b Q_K - w)\ell_b\right), \tag{21}$$

where $Q_K = B(\lambda_b^{1-\omega}x^{\eta_b}\ell_b^{\nu_b-1})$ is treated by firms as an exogenous constant in the steady state (Jones, 2005a; Romer, 1990) – though it will be determined by the respective model variables in equilibrium. Analogously, labor augmenting R&D firms maximize:

$$\max_{\ell_a} \left(p_a\dot{\lambda}_a - w\ell_a\right) = \max_{\ell_a}\left((p_a Q_L - w)\ell_a\right), \tag{22}$$

where $Q_L = A(\lambda_a\lambda_b^{\phi}x^{\eta_a}\ell_a^{\nu_a-1})$ is treated as exogenous.

Free entry into both R&D sectors implies $w = p_b Q_K = p_a Q_L$. Purchase of a patent entitles the holders to a per-capita stream of profits equal to π_K and π_L, respectively. While the production of any labor augmenting varieties lasts forever, there is a constant rate d at which production of capital-intensive varieties becomes obsolete. This effect is external to patent holders and thus is not strategically taken into account when accumulating the patent stock.[20]

3.3.6. Externality term

There is also an (optional) externality term in the capital's equation of motion. Motivated by León-Ledesma and Satchi (2017), we allow for a non-negative cost of adopting new labor-augmenting technologies: since workers (as opposed to machines) need to develop skills compatible with each new technology, it is assumed that there is an external capital cost of such technology shifts (training costs, learning-by-doing, etc.). Based on a detailed micro-founded derivation available in our working paper version (Growiec et al., 2015) we posit that new capital investments are diminished by $\zeta z_a L$, where $\zeta \geq 0$ and $z_a = g\lambda_a \frac{\pi}{\pi_0}$, and thus $\frac{z_a}{k} = \frac{g\lambda_b}{x}\frac{\pi}{\pi_0}$.

3.3.7. Equilibrium

We define the *decentralized equilibrium* as the collection of time paths of all the respective quantities: c, ℓ_a, ℓ_b, k, λ_b, λ_a, Y_K, Y_L, $\{X_{Ki}\}$, $\{X_{Li}\}$ and prices r, w, p_K, p_L, $\{p_{Ki}\}$, $\{p_{Li}\}$, p_a, p_b such that: (1) households maximize discounted utility subject to their budget constraint; (2) profit maximization is followed by final-goods producers, aggregators and producers of capital and labor-intensive intermediate goods, and capital and labor augmenting R&D firms; (3) the labor market clears: $L_a + L_b + L_Y = (\ell_a + \ell_b + \ell_Y)L = L$; (4) the asset market clears: $V = \nu L = K + p_a\lambda_a + p_b\lambda_b$, where assets have equal returns: $r - \delta = \frac{\pi_L}{p_a} + \frac{\dot{p}_a}{p_a} = \frac{\pi_K}{p_b} + \frac{\dot{p}_b}{p_b} - d$; and, finally (5), such that the aggregate capital stock satisfies

$$\dot{K} = Y - C - \delta K - \zeta z_a L, \qquad \Leftrightarrow \qquad \dot{k} = y - c - (\delta + n)k - \zeta z_a. \tag{23}$$

3.4. Solving for the decentralized allocation

When solving for the decentralized allocation, we first solve analytically for the BGP of our endogenous growth model and then linearize the implied dynamical system around the BGP.

3.4.1. Balanced growth path

Since Uzawa (1961) we have known that any neoclassical growth model can exhibit balanced growth only if technical change has a purely labor-augmenting representation or if production is Cobb–Douglas.[21] This conclusion holds for our model too. Hence, once we presume a CES production function, the analysis of dynamic consequences of technical change, which is not purely labor augmenting, must be done outside the BGP.

Along the BGP, we obtain the following growth rate of key model variables:

$$g = \hat{\lambda}_a = \hat{k} = \hat{c} = \hat{y} = A(\lambda_b^*)^{\phi}(x^*)^{\eta_a}(\ell_a^*)^{\nu_a}, \tag{24}$$

where stars denote steady-state values and, as before, $\hat{q} = \dot{q}/q$ etc.

Hence, ultimately long-run growth is driven by labor augmenting R&D. This can essentially be explained by the fact that labor is the only non-accumulable factor in the model, it is complementary to capital along the aggregate production function, and the labor augmenting R&D equation is linear with respect to λ_a. The following variables are constant along the BGP: y/k, c/k, Y_K/Y, Y_L/Y, ℓ_a, ℓ_b and λ_b. Stability of the last variable at the BGP signifies that, unsurprisingly, there is no capital augmenting technical change along the BGP.

[20] In other words, by solving a static optimization problem, capital augmenting R&D firms do not take the dynamic (external) obsolescence effect into account.

[21] Irmen (2016) generalizes Uzawa's result to growth models allowing aggregate intermediate expenses to follow a different technology than output.

3.4.2. Euler equations

Calculations imply that the decentralized equilibrium is associated with the following Euler equations describing the first-order conditions:

$$\hat{c} = \frac{\varepsilon\pi \frac{y}{k} - \delta - \rho}{\gamma}, \tag{25}$$

$$\varphi_1 \hat{\ell}_a + \varphi_2 \hat{\ell}_b = Q_1, \tag{26}$$

$$\varphi_3 \hat{\ell}_a + \varphi_4 \hat{\ell}_b = Q_2, \tag{27}$$

where

$$\varphi_1 = \nu_a - 1 - (1-\xi)\pi \frac{\ell_a}{\ell_Y} \; ; \; \varphi_4 = \nu_b - 1 - (1-\xi)\pi \frac{\ell_b}{\ell_Y}$$

$$\varphi_2 = -(1-\xi)\pi \frac{\ell_b}{\ell_Y} \; ; \; \varphi_3 = -(1-\xi)\pi \frac{\ell_a}{\ell_Y}$$

$$Q_1 = -\varepsilon\pi \frac{y}{k} + \delta + \hat{\lambda}_a \frac{\ell_Y}{\ell_a} - \phi\hat{\lambda}_b + ((1-\xi)\pi - \eta_a)\hat{x}$$

$$Q_2 = -\varepsilon\pi \frac{y}{k} + \delta + \hat{\lambda}_a + (\hat{\lambda}_b + d)\left(\frac{\pi}{1-\pi}\frac{\ell_Y}{\ell_b}\right) - \hat{\lambda}_b(1-\omega) - d + ((1-\xi)\pi - \eta_b)\hat{x}.$$

A sufficient condition for all transversality conditions to be satisfied is that $(1-\gamma)g + n < \rho$.

3.4.3. Steady state of transformed system

To analyze the properties of the dynamic system around the BGP, the Euler equations and dynamics of state variables have been rewritten in terms of *stationary* variables which are constant along the BGP, i.e., in coordinates: $u = (c/k), \ell_a, \ell_b$, x, λ_b, with auxiliary variables $z = (y/k), \pi, g$. The steady state of the transformed system satisfies:

$$g = \hat{\lambda}_a = \hat{k} = \hat{c} = \hat{y} = A(\lambda_b^*)^{\phi}(x^*)^{\eta_a}(\ell_a^*)^{\nu_a} \tag{28}$$

$$\gamma g + \rho = r - \delta \tag{29}$$

$$g = z - \zeta\frac{z_a}{k} - u - (\delta + n) \tag{30}$$

$$d = B\left(\lambda_b^{-\omega} x^{\eta_b} \ell_b^{\nu_b}\right) \tag{31}$$

$$g\frac{\ell_Y}{\ell_a} = r - \delta \tag{32}$$

$$g = r - \delta + d\left(1 - \frac{\pi}{1-\pi}\frac{\ell_Y}{\ell_b}\right) \tag{33}$$

$$r = \varepsilon\pi z \tag{34}$$

$$\frac{\pi}{\pi_0} = \left(\frac{\lambda_b}{\lambda_{b0}}\right)^{\xi}\left(\frac{z}{z_0}\right)^{-\xi} \tag{35}$$

$$\frac{z}{z_0} = \frac{\lambda_b}{\lambda_{b0}}\left(\pi_0 + (1-\pi_0)\left(\frac{x_0}{x}\frac{\ell_Y}{\ell_{Y0}}\right)^{\xi}\right)^{1/\xi}. \tag{36}$$

All further analysis of the decentralized allocation will be based on the numerical linearization of the 5-dimensional dynamical system of equations (3), (23) and (25)–(27), in coordinates $u = (c/k), \ell_a, \ell_b, x, \lambda_b$, taking (2) as given, around the unique steady state of the de-trended system (and thus, around the unique BGP of the model in original variables).

Table 3
Baseline calibration: pre-determined parameters.

Parameter		Value	Source/Target
Preferences			
Inverse Intertemporal Elasticity of Substitution	γ	1.7500	Barro and Sala-i-Martin (2003)
Time Preference	ρ	0.0200	Barro and Sala-i-Martin (2003)
Income and Production			
GDP Per-Capita Growth	g	0.0171	Geometric average
Population Growth rate	n	0.0153	Geometric average
Labor in Aggregate Production	ℓ_{Y0}, ℓ_Y^*	0.5934	$\frac{0.5(\gamma+\frac{\rho}{g})}{1+0.5(\gamma+\frac{\rho}{g})}$
Capital Productivity	z_0, z^*	0.3450	Geometric average
Consumption-to-Capital	u^*	0.2199	Geometric average
Capital Income Share	π_0, π^*	0.3260	Arithmetic average
Depreciation	δ	0.0600	Caselli (2005)
Factor Substitution Parameter	ξ	−0.4286	$\Rightarrow \sigma = 0.7$, Klump et al. (2007)
Net Real Rate of Return	$r^* - \delta$	0.0499	$r^* - \delta = \gamma g + \rho$
Substitutability Between Intermediate Goods	ε	0.9793	$\varepsilon = \frac{r^*}{\pi^* z^*}$
R&D Sectors			
R&D Duplication Parameters	$\nu_a = \nu_b$	0.7500	See text
Technology-Augmenting Terms	$\lambda_{a0}, \lambda_{b0}$	1.0000	See text
Technology-Augmenting Terms	λ_b^*	1.0000	$\lambda_b^* = \lambda_{b0} \frac{z^*}{z_0} \left(\frac{\pi^*}{\pi_0}\right)^{\frac{1}{\xi}}$
Labor Input in R&D sectors	ℓ_a^*, ℓ_b^*	0.2033	$\ell_a^* = \ell_b^*$ for $\ell_a^* + \ell_b^* = 1 - \ell_Y^*$
Effective Capital-Labor ratio†	x_0, x^*	61.7900	$x^* = x_0 \frac{\ell_Y^*}{\ell_{Y0}} \left(\frac{1}{1-\pi_0} \left(\frac{z^*}{z_0} \frac{\lambda_{b0}}{\lambda_b^*} \right)^{\xi} - \frac{\pi_0}{1-\pi_0} \right)^{-1/\xi}$

Notes: † $x_0 = \frac{\lambda_{b0} k_0}{\lambda_{a0}} = 61.79$.

4. Model calibration

The calibration (see Table 3) follows five steps. First, several "deep" parameters are pre-determined by taking values stemming from the literature: the (inverse) intertemporal elasticity of substitution, the rate of time preference, and depreciation. These first two parameters turn out to be key to generating limit cycles, for a given technology process.

Second, we assign CES normalization parameters to match US in-sample long-run averages. This implies an average labor share of $1 - \pi_0 = 0.66$. Following many empirical studies (Chirinko, 2008; Klump et al., 2007; Oberfield and Raval, 2014) we calibrate factors to be gross complements.[22] However, since some other studies (e.g., Karabarbounis and Neiman, 2014; Piketty, 2014) rely on gross substitutes, we also consider this in our robustness exercises. Third, we assume that a range of long-run averages from US data correspond to the model's BGP. Doing so allows us to calibrate the rates of economic and population growth, capital productivity, and the consumption-to-capital ratio.

Next, with this in hand, four identities included in the system (28)–(36) yield the calibration of other parameters in a model-consistent manner: $\ell_Y^*, r^*, \lambda_b^*, x^*$ and ε. Final production employment is also set in a model-consistent manner. To start with, we agnostically assume the share $1 - \ell_Y^*$ is split equally between employment in both R&D sectors. For the model-consistent value of ℓ_Y^*, this formula leads to values close to those typically considered under the non-routine cognitive occupational group (e.g., using BLS data, Jaimovich and Siu, 2012 show this ratio to be between 29% and 38% (over 1982–2012); see also Autor et al., 2003).[23]

Regarding the duplication externalities in factor augmenting R&D, as earlier stated, the literature typically considers a unique R&D duplication externality. Following Jones and Williams (2000)'s single aggregate value, we set $\nu_a = \nu_b = 0.75$.[24] The technology augmenting term λ_b^* is set in a model-consistent manner.

The final step is to assign values to the remaining parameters, in particular the technological parameters of R&D equations. We do this by solving the four remaining equations in the system (28)–(36) with respect to the remaining parameters, see Table 4. All these parameters are within admissible ranges. For instance, Pessoa (2005) estimates values for the obso-

[22] Quoted studies rely on consistently measured data exploiting time-series variation. Still, León-Ledesma et al. (2010) found that even studies based on long samples reported estimates of the US substitution elasticity below one. Arrow et al. (1961) found an aggregate elasticity over 1909–1949 of 0.57. Oberfield and Raval (2014) report their average estimate of the aggregate elasticity at 0.7 based on a large firm-level data set from US manufacturing, with substantial cross-sectional variation. On the other hand, literature based predominantly on cross-country variation (e.g., Karabarbounis and Neiman, 2014; Piketty and Zucman, 2014), tends to imply gross substitutability.

[23] Counting the number of scientists, researchers, teachers and even patents and expenditures has long been recognised as a crude proxy for research activity. (See the "Oslo Manual" (OECD/Eurostat, 2005) for a discussion of the various R&D types, and measurement issues.) Thus, we might also choose to interpret the ℓ_a^* and ℓ_b^* values as a correction for the managerial and entrepreneurial input to production as well as learning-by-doing on the side of employees; when new technologies are implemented in production, they require significant effort and/or reorganization of the workplace, which might be considered to show up as R&D in our simplified model. Similarly, it may capture non-routine and analytical tasks in the employment spectrum which do not necessarily show up in formal research-intensive job definitions

[24] This can also be justified as an average of the original constant-returns-to-scale parametrization of $\nu = 1$ (Romer, 1990) and the evidence of $\nu \approx 0.5$ provided by Pessoa (2005).

Table 4
Baseline calibration: additional parameters.

Parameter		Value
Labor augmenting R&D		
Unit productivity	A	0.02
Capital-Labor in R&D exponent	η_a	0.24
Capital augmenting R&D		
Unit productivity	B	0.16
Capital-Labor in R&D exponent	η_b	0.13
Degree of decreasing returns	ω	0.50
Obsolescence rate	d	0.08
Spillover from capital to labor augmenting tech. change	ϕ	0.30
Technology choice externality	ζ	115.28

Table 5
Dynamics around the BGP under the baseline calibration.

Dynamic metric	
Pace of Convergence* (% per year)	6.3%
Length of Full Cycle† (years), $\mathbf{L}^c$	52.6
Labor Share Cyclicality	+

Note: Pace of convergence reflects by how many % of the current distance does the system's distance to the BGP decrease per annum. *computed as $1 - e^{rr}$ where $rr < 0$ is the real part of the largest stable root; † computed as $\mathbf{L}^c = 2\pi/ir$ where $ir > 0$ is the imaginary part of two conjugate stable roots (if they exist). A '+' indicates pro-cyclicality.

lescence parameter between 0 and 15%; our endogenously determined value is thus centered in that range. Comparing $\eta_a = 0.24$ with $\eta_b = 0.13$ signifies that, first of all, lab equipment (effective capital augmentation of the R&D process) assuredly matters for R&D productivity, and second, that it is relatively more important for inventing new labor-augmenting technologies than capital-augmenting ones. Moreover, with $\phi = 0.3$, labor-augmenting R&D the ultimate engine of long-run growth is substantially reinforced by spillovers coming from the capital-augmenting R&D sector. On the other hand, $\omega = 0.5$ means that the scope for capital-augmenting R&D is quite strongly limited by decreasing returns. Given this benchmark calibration, the steady state is a saddle point.

5. Oscillatory model dynamics

Given our baseline calibration, the decentralized allocation of the model exhibits endogenous, dampened oscillations of the labor share and other de-trended model variables, see Table 5. These are long swings, similar to the one observed throughout the 20th century in the US (recall Fig. 3) and elsewhere (Appendix C.2) rather than business-cycle fluctuations.[25]

Moreover, we also obtain quantitative predictions on cyclical co-movements.[26] It turns out that all variables except for the consumption-capital ratio $u = c/k$ oscillate when converging to the steady state, with the same frequency of oscillations.[27] The labor share π (as long as capital and labor are gross complements, $\xi < 0$), capital-augmenting R&D employment ℓ_b and employment in production ℓ_Y are always pro-cyclical, whereas labor-augmenting R&D ℓ_a, the level of capital-augmenting technology λ_b and the effective capital-labor ratio x are counter-cyclical. More specifically, in the long-run limit the amplitude of oscillations of the labor share π is ca. 62% relative to output per unit of capital, y/k. For other variables, respective relative amplitudes are: 26% for ℓ_a, 12% for ℓ_b, 14% for ℓ_Y, and 150% for λ_b.[28]

These features of cyclical co-movement align well with the available empirical evidence for the US medium-term cycle (though some of them could only be verified in the future, once reliable data on variables such as ℓ_a and ℓ_b become avail-

[25] Bear in mind that, by design, our method of analysis does not allow us to test whether the model succeeds in matching the absolute *amplitude* of cycles. This is because the analysis is carried out based on a system that has been linearized around the BGP. Then the amplitude of oscillations is just a matter of initial conditions: if the system is started far away from the BGP then the amplitude must be large, and conversely, it will be small if the system is started close to the BGP. We do not have sufficient empirical information on the appropriate initial conditions, and moreover, the distance from BGP must be kept manageably small in the analysis for the linearized system to be a sufficiently good approximation of the full nonlinear model.

[26] This is done by inspecting the eigenvector associated with the largest stable root of the Jacobian of the system at the steady state.

[27] In our real deterministic model, that correlation is 1 (compared to 0.6 in the data, recall Table 2). Introducing additional, for example nominal, frictions would be one reconciliation device.

[28] Recall that relative amplitudes can be computed only for de-trended variables which oscillate around their respective steady states.

able). In particular, the US labor share is indeed pro-cyclical over the medium-to-long run (despite its counter-cyclicality along the business cycle).

5.1. Emergence of limit cycles

Could one plausibly expect limit cycle behavior of the labor share (and as a consequence, the growth rate and other endogenous variables) in this model? At the baseline calibration, the answer is no because (as just noted) the model exhibits *dampened* cycles, i.e., oscillatory convergence to the BGP. On the other hand, we know that limit cycles can be generated by Hopf's bifurcation theorem (Feichtinger, 1992; Kuznetsov, 2004). This states that if, when exploring the support of one of the model parameters, real parts of two stable conjugate roots of the system transversally cross zero, then the steady state loses its local stability.

The ensuing bifurcation can be either *supercritical* or *subcritical*. In the supercritical case, the steady state is first a global attractor, and when it becomes locally unstable, a stable limit cycle is created around it. In the subcritical case, the stable steady state is only a local attractor surrounded by an unstable limit cycle, and when it loses stability, it becomes globally repelling. Whether the Hopf bifurcation is supercritical or subcritical, depends on the sign of the first Lyapunov coefficient at the bifurcation point.[29] The question then is, does any of these situations appear in our case, and if so, do they occur around an empirically plausible parameter set?

To answer this, we carried out the following "multi-calibration" exercise (or BGP preserving sensitivity analysis). We computed the eigenvalues of the system around the steady state for different values of a particular parameter, making sure that whatever assumption on its value is made, other pre-determined parameters are held at their baseline values whereas other "free" parameters are re-calibrated in a way that the model always remains in accordance with (data-consistent) BGP characteristics. In this way, we identified model parameterizations leading to various types of local dynamics around *the same* BGP. Having identified the point of Hopf bifurcation, i.e., the parametrization under which the eigenvalues of the system around the steady state transversally cross zero, we computed the first Lyapunov coefficient of the system at this point.[30]

Parameters not pinned down by the data are the key ones we investigate as regards the model's quantitative and cyclical behavior. These are essentially $\gamma, \rho, \delta, \xi, \frac{\ell_a^*}{\ell_a^*+\ell_b^*}$ and ν_a, ν_b (respectively: inverse intertemporal substitution elasticity, discount factor, depreciation rate, aggregate production elasticity parameter, share of labor augmenting R&D at the BGP, two duplication parameters).[31]

Fig. 4 illustrates the results of this quantitative exercise with respect to ρ (time preference), γ (inverse elasticity of intertemporal substitution) and $\frac{\ell_a^*}{\ell_a^*+\ell_b^*}$ (the share of labor augmenting R&D at the BGP). Different calibrations of the relevant parameter are marked on the horizontal axis, whereas the resulting real and imaginary parts of the eigenvalues of the dynamical system (linearized around the BGP) are plotted along the vertical. Positive (negative) real parts imply divergence (convergence). The presence of non-zero imaginary parts indicates oscillatory dynamics. Specifically, we find that:

(i) when the discount factor falls below its bifurcation value of $\rho^{\text{bif}} = 0.0061$, a *subcritical* Hopf bifurcation is obtained, with the first Lyapunov coefficient
$l_1(u^*, \ell_a^*, \ell_b^*, x^*, \lambda_b^*) = 1.1 \times 10^{-6} > 0$,

(ii) when the inverse intertemporal elasticity of substitution falls below its bifurcation value of $\gamma^{\text{bif}} = 0.942$, a *supercritical* Hopf bifurcation is obtained, with
$l_1(u^*, \ell_a^*, \ell_b^*, x^*, \lambda_b^*) = -8.1 \times 10^{-7} < 0$,

(iii) when the steady-state ratio of labor augmenting R&D to total R&D employment falls below $\left(\frac{\ell_a^*}{\ell_a^*+\ell_b^*}\right)^{\text{bif}} = 0.425$, a *subcritical* Hopf bifurcation is obtained, with
$l_1(u^*, \ell_a^*, \ell_b^*, x^*, \lambda_b^*) = 5.0 \times 10^{-7} > 0$.

It follows that stable limit cycles can appear in this model when individuals, *ceteris paribus*, are sufficiently willing to intertemporally substitute consumption (low γ). In such a case, their consumption smoothing motive is too weak to compensate for the fluctuations in the labor share as well as the degree of capital augmentation λ_b, caused by the simultaneous arrivals of capital augmenting developments (subject to gradual depreciation) and changes in the pace of (continued) labor augmenting technical change. Although both these changes are productivity-enhancing, they are destabilizing due to their opposing impacts on the labor share. Long swings in economic activity are then perpetuated.

[29] More precisely, in the multi-dimensional case with which we are dealing here, at the point of a Hopf bifurcation the steady state ceases to be a stable focus along the stable manifold, and becomes a repelling focus instead. The considered limit cycles around the steady state are also located in the same manifold.

[30] Apart from the eigenvalues and eigenvectors, computing the first Lyapunov coefficient also requires a numerical approximation of second and third order partial derivatives of the system at the steady state (Kuznetsov, 2004). Matlab codes for the "multi-calibration" exercise as well as computing the first Lyapunov coefficient are available from the authors upon request.

[31] Although results for parameters (δ, η_a, ν_b, ν_a) do not in themselves produce qualitative changes in model dynamics (details available on request).

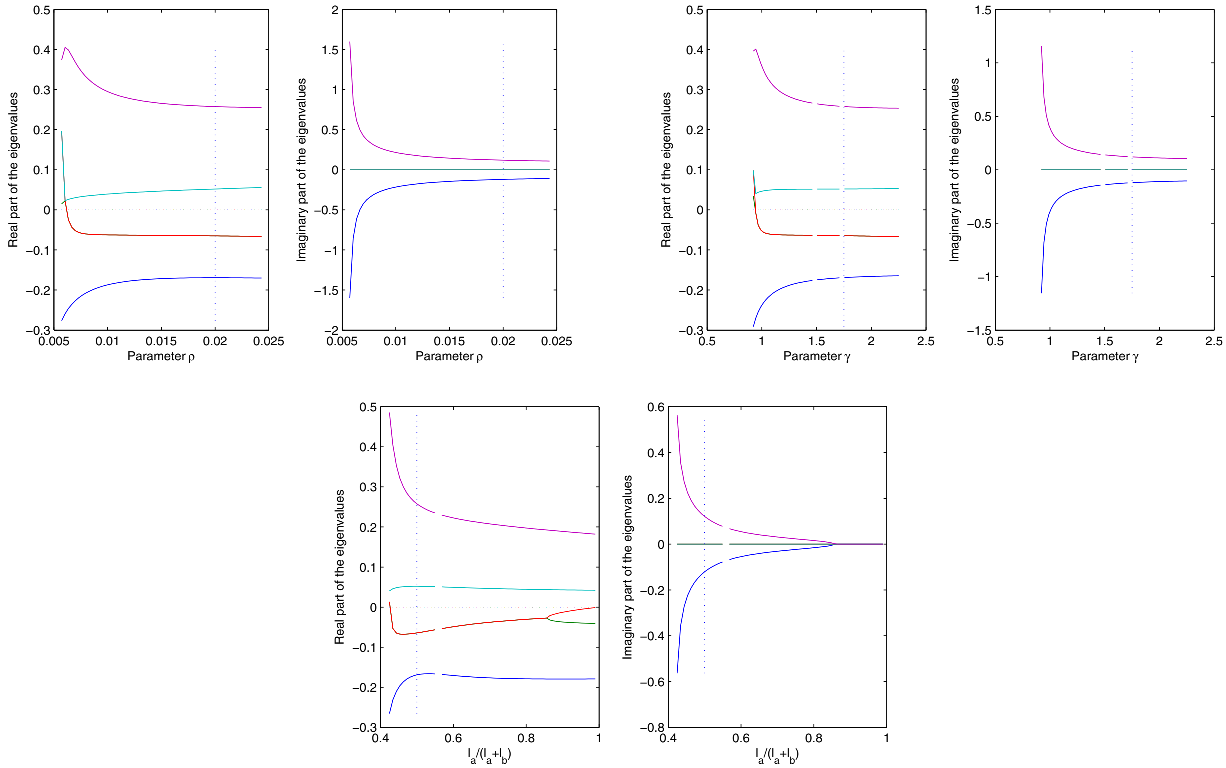

Fig. 4. Emergence of limit cycles via Hopf bifurcations. Note: vertical lines indicate the baseline calibration.

Moreover, note that all three bifurcation values are in the ballpark of empirically plausible ones. To generate a stable limit cycle in factor shares (as well as other model variables), it suffices that the household is just slightly more willing to intertemporally substitute consumption than under log preferences.

And although the bifurcation value of the time preference rate is rather low in the baseline case, it should be noted that with $\gamma = 1$ (log preferences), the bifurcation value with respect to ρ appears already around 0.019, which is very close to its baseline value.

5.2. Node-focus bifurcations

In addition to Hopf bifurcations, another interesting type of sudden changes in model dynamics can be observed: a "node-focus" bifurcation. If, manipulating one of the model parameters, two stable real eigenvalues collide and become complex conjugates, then the pattern of steady-state convergence changes from monotonic to oscillatory (see Fig. E.6 in the appendix). These arise for manipulations in parameters ξ (related to the substitution elasticity) and η_b (the capital-labor exponent capital augmenting R&D).

When ξ becomes sufficiently large (and already above zero implying gross factor substitutability), then the imaginary parts of two conjugate stable roots hit zero, so that oscillatory dynamics are eliminated. Under gross complementarity, the magnitude of input complementarity generally increases the frequency of observed oscillations.[32]

Oscillatory dynamics also prevail only if η_b is sufficiently low, and the lower it is, the higher the oscillation frequency. A fully analogous result is also found when manipulating $\frac{\ell_a^*}{\ell_a^*+\ell_b^*}$ (the share of labor augmenting R&D in total R&D at the BGP). When this share is sufficiently high (above 90%), dampened oscillations disappear in favor of monotonic convergence.

5.3. On the value of the elasticity of substitution

Whether the substitution elasticity exceeds unity or not is important for the prevalence of oscillatory convergence to the BGP and emergence of Hopf bifurcations. Departing from our benchmark, we now repeat our analysis assuming $\sigma = 1.25$ following Karabarbounis and Neiman (2014), see Table 6. We still obtain the oscillatory convergence result, although the

[32] Finally, as far as manipulations in ξ are concerned, we also observe a further phenomenon. Namely, in the range of gross substitutability there exists a critical value of ξ when a real root switches its sign. At this point a *generalized saddle-node bifurcation* appears, due to which the steady state loses its stability (but without creating any limit cycles).

Table 6
Dynamics around the BGP under the baseline calibration for $\xi = 0.2$ ($\sigma = 1.25$).

Dynamic metric	
Pace of convergence (% per year)	5.2%
Length of full cycle (years), $\mathbf{L}^c$	144.0
Labor share cyclicality	–

A '–' indicates counter-cyclicality.

Table 7
Dynamic model properties: duplication externalities.

Base calibration with ...	ν_b/ν_a	Pace of convergence (% Per Annum)	Cycle length (Years)	Conditional on $\nu_a, \nu_b^\dagger$ γ^{bif}	ρ^{bif}
Symmetric duplication					
$\nu_a = \nu_b = 0.1$	1	4.84%	Monotonic	0.9237	0.0059
$\nu_a = \nu_b = 0.5$	1	5.58%	97.62	0.9534	0.0064
$\nu_a = \nu_b = 0.75$	1	6.30%	52.65	0.9430	0.0062
$\nu_a = \nu_b = 0.9$	1	9.44%	29.45	No Hopf	No Hopf
Asymmetric duplication					
$\nu_a = 0.9, \nu_b = 0.1$	0.1	4.32%	Monotonic	No Hopf	No Hopf
$\nu_a = 0.9, \nu_b = 0.5$	0.6	9.93%	67.35	No Hopf	No Hopf
$\nu_a = 0.9, \nu_b = 0.9$	1	9.44%	29.45	No Hopf	No Hopf
$\nu_a = 0.5, \nu_b = 0.9$	1.8	4.53%	54.55	1.0794	0.0086
$\nu_a = 0.1, \nu_b = 0.9$	9.0	4.05%	72.31	1.0864	0.0090

Note: The labor share and the per-capita growth rate at the BGP in the decentralized allocation are exactly matched to the long-run US averages (0.6739, 0.0171 respectively) for each parametrization, and thus are not shown. † Following a BGP-preserving sensitivity analysis. "Monotonic" indicates monotonic convergence to the steady state; otherwise there are dampened oscillations along the convergence path towards the steady state. "No Hopf" indicates that for given ν_a, ν_b, Hopf bifurcations cannot be obtained for any γ or ρ.

pace of convergence is now somewhat shorter than before, but the cycle length is a highly counterfactual 144 years. In addition, in contrast to our earlier empirical and model-based analysis (Sections 2 and 5), the labor share is now counter-cyclical. Hence, the case for our original assumption of gross complements seems to be strengthened.

Finally, results are now more in favor of an emergence of limit cycles. Bifurcation values are now closer to our baseline calibration, and the bifurcation is now supercritical in both cases (not just in the case of γ).

(i) Bifurcation with respect to the discount rate: $\rho^{bif} = 0.00766$, $l_1 = -2.45 \times 10^{-7} < 0 \Rightarrow$ supercritical bifurcation $\Rightarrow$ emergence of limit cycle.
(ii) Bifurcation with respect to the intertemporal elasticity of substitution in consumption: $\gamma^{bif} = 1.04092$, $l_1 = -7.31 \times 10^{-7} < 0 \Rightarrow$ supercritical bifurcation $\Rightarrow$ emergence of limit cycle.

5.4. Labor share cycles and duplication externalities

We now isolate the effect of variations in the magnitude of duplication externalities on model dynamics. We scrutinize the impact of these particular parameters, and not others, for three reasons. First, based on the literature we infer substantial uncertainty in their value. Second, our detailed analysis, revealed that the model dynamics depend critically on the value of the latter of these two parameters, ν_b, but less so on other uncertain parameters (details available on request). Finally, given our interest in the labor income share, it makes sense to concentrate on parameters whereby endogenous R&D growth is directly affected by labor flows.

Table 7 looks at the consequences of varying the ν's (symmetrically and asymmetrically) in terms of the implied pace of convergence to the BGP and cycle length. These variations around the baseline typically lead to dampened cycles, and occasionally to monotonic convergence. The final column takes all parameters as given, including the particular ν pairings, and varies γ and ρ separately until a Hopf bifurcation (if it exists) is identified.

5.4.1. Symmetric duplication

Our results suggest cycle length, $\mathbf{L}^c$, is increasing with the magnitude of duplication externalities (i.e., it is decreasing with $\nu_a = \nu_b$):

$$\left.\frac{\partial \mathbf{L}^c}{\partial \nu_a}\right|_{\nu_a=\nu_b} < 0.$$

Our baseline case ($\nu_a = \nu_b = 0.75$), implies a cycle length of 52 years. At the extremes, however, this can change to, e.g., over 98 years or around 30 years (recalling Section 2.2, this is the dominant frequency of medium-term oscillations present in the US data).

The intuition behind this result is the following. As duplication externalities fall (the ν values rise), the return from labor flows into each of the R&D sectors becomes higher and the gestation period for new ideas to "come on-stream" is accordingly reduced; thus cycle length shortens.

At $\nu_a = \nu_b = 0.75$ Hopf bifurcations arise if the households' consumption smoothing motive is less strong than in the baseline ($\gamma = 0.94$ vs. 1.75) or if the society becomes more patient ($\rho = 0.006$ vs. 0.02). Again, this makes sense. If the representative household gives a high weight to future generations (low ρ), it is willing to invest substantial resources in physical capital and both R&D types. R&D, however, incurs the cost of obsolescence of capital augmenting technologies. Furthermore, under CES technology, this shift in labor allocation affects the labor share, and also propagates via the mutual R&D spillovers and the effective capital-labor terms. If, ultimately, the consumption-smoothing motive is weak enough (low γ), these destabilizing effects are not countered by lowering R&D employment or savings, which leads to endogenous cycles.

5.4.2. *Asymmetric duplication*

As before, with weak duplication externalities in labor augmenting R&D (large ν_a), there is no limit cycle for any corresponding capital augmenting value. Again this is intuitive. The balanced growth path is driven by labor augmenting technologies alone. The more smoothly labor augmenting ideas are being produced, the closer at any point is the economy to its balanced growth path, and thus less amenable to limit cycles.

A corollary of this can be seen when $\nu_b/\nu_a > 1$ (representing the case of "labor-biased duplication", when duplication externalities are stronger in labor augmenting R&D). Here the economy is far away from balanced technical growth in the sense that the capital augmenting R&D sector is less constrained by duplication than its labor equivalent. The possibility for waves of innovation and thus excessively fast replacement and obsolescence of existing ideas is then more likely to produce exaggerated cycles. In such cases, the consumption smoothing preferences consistent with a limit cycle are still below the baseline value but now closer to and marginally above log preferences.

Note finally that although our study does not intend to match the frequency of medium-to-long swings exactly, we still view the R&D-based endogenous growth model with CES production as a viable explanation for the hump-shaped trend of the labor share observed in the US throughout the twentieth century. In fact, if one argued that in our 83-year time series of the US labor share, we observe only a half of a full swing, then the model could match that exactly if the imaginary parts of stable roots were around 0.04 which could be obtained e.g., for somewhat lower duplication parameters ν_a and ν_b, a higher capital-labor R&D exponent η_b or lower η_a, etc.

6. Conclusions

The contribution of the article has been (i) to document that the labor share exhibits medium-to-long run, pro-cyclical swings suggestive of a long cycle, and (ii) assess the extent to which an endogenous growth model can account for those regularities.

The model implies oscillatory behavior of factor shares along the convergence path to the BGP. This is due to the interplay between arrivals of capital augmenting developments (subject to gradual depreciation) and changes in the pace of (continued) labor augmenting technical change. The model delivers plausible implications regarding the co-movement of other variables along the labor share swings. Under certain parametrizations, the model gives rise to Hopf bifurcations, leading to self-sustaining limit cycles of reasonable length. These parameterizations are within an empirically-plausible neighborhood, such as to suggest that the framework considered *naturally* gives rise to endogenous factor cycles, rather than in identifying special cases.

The formalization of endogenous cycles in factor shares is an important insight since mainstream economists have either emphasized the historical stability of factor shares, or else focused on particular episodes of drifting shares. However, long data sets which are now becoming available uncover that, before their recent decline, shares have often in fact been trending upward for decades (as we might expect of a bounded series). Our model features endogenous mechanisms able to account for both tendencies. For competing theories, it is a challenge to do so.

Our framework also points to additional lines of inquiry. First, typically in endogenous growth models a research subsidy is recommended to align the decentralized allocation with the socially optimal one.[33] In our framework it may be worth investigating whether such a subsidy would (or should) also have cyclical characteristics, or whether such a subsidy could be feasibly created. If that subsidy were skewed more towards one factor augmenting technology than another, what would be the distributional and cyclical consequences?

Second, discussion of labor share declines have often gone hand-in-hand with those on 'globalization' which is often interpreted as a widening of the pool of available labor. But it could also relate to the extent to which ideas can be protected

[33] Although note that in variety expansion models the decentralized steady-state growth rate may well exceed the socially optimal one if we control for the gains from specialization in the production functions where the expansion of varieties matters (see Bénassy, 1998).

and accessed, i.e., to duplication externalities. Such externalities played a prominent role in our analysis. Thus our model can provide a platform to discuss those broader issues as rooted in technological developments.

Finally, we have refrained from any welfare-based statements. The model laid out here is insufficient for such analysis. But one could envisage analysis aimed at defining whether the cycles (be they convergent or sustained) are or are not welfare enhancing, and thus whether public intervention is warranted. Interestingly Atkinson (2015) lists a number of proposals for reducing inequality trends, the first of which is that "The direction of technical change should be an explicit concern of policy-makers". Political agents, moreover, may take a view on the desirability of economic volatility and the length of and distributional consequences of economic cycles. We leave these for subsequent discussion.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.jedc.2017.11.007.

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