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HUMAN CAPITAL, AGGREGATION, AND GROWTH

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Human capital is embodied in people of different generations whose lifetimes are finite. We show that the finiteness of people's lives precludes human capital accumulation from driving long-run aggregate economic growth unless sufficiently strong externalities from aggregate human capital are introduced. Two possible channels for carrying forward such externalities are (i) knowledge spillovers and (ii) public education spending. Our findings shed new light on the foundations of the Uzawa–Lucas growth model. We also show that the cross-sectional Mincer equation, generated by a linear human capital accumulation equation at the individual level, does not carry forward to aggregate data.

Keywords: Human Capital Accumulation, Aggregation across Vintages, Externalities, Balanced Growth

1. INTRODUCTION

Human capital, that is, "skills embodied in a worker" [Barro and Sala-i-Martin (1995, p. 172)] has a number of important properties. First of all, it is embodied, rival, and excludable. Moreover, because it is embodied in people, it is lost upon their deaths. It is also not directly transferable across generations: newborn babies do not inherit the human capital of their parents automatically; they have to learn the skills themselves, while parents and teachers can only offer guidance and help.

Thus, in thinking about aggregate human capital accumulation, it is necessary to go beyond the observation that it is accumulated through schooling, training, and on-the-job learning; demographics following from people's finite lifetimes ought to be accounted for as well. Furthermore, because people die and are born at different moments in time, and because heterogeneity in ages generates heterogeneity in human capital levels, the aggregation procedure should be based on an explicit vintage structure of human capital [cf. Boucekkine et al. (2002)].

The objective of this paper is thus to carry out such an aggregation procedure. Based on its outcomes, several propositions will be derived emphasizing the

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decisive impact of aggregation on the ability of human capital accumulation to drive long-run growth.

For tractability and transparency of our principal results, we shall ignore intracohort heterogeneity. The only source of heterogeneity will thus be the age of individuals, or equivalently, the cumulative amount of education they have received in their lives.

The contribution of this article to the literature is threefold. First, we find that under finite lifetimes and in the absence of human capital externalities, *the human capital accumulation sector alone is not able generate long-run balanced growth*, even if the schooling technology is linear at the individual level. We derive this proposition from a simple generic model (the appendix shows that it is actually robust to a number of extensions). This finding contrasts sharply with the assumptions of the well-known Uzawa–Lucas model [Uzawa (1965); Lucas (1988)], where growth is driven by human capital accumulation by infinitely lived individuals (or alternatively, by accumulation of disembodied "human capital").

There exists a way to reconcile the Uzawa–Lucas model with finite lifetimes, though. As Lucas (1988) acknowledged, "it would take some work to go from a human capital technology of the [linear] form . . . , applied to each finite-lived individual . . . , to this same technology applied to an entire infinitely-lived typical household or family" [Lucas (1988, p. 19)]. This work turned out to be largely conceptual rather than just technical, and it motivated the second contribution of this article.

The second contribution is to show that the prediction of *long-run balanced growth driven by human capital accumulation can be rescued by introducing externalities from aggregate human capital* into human capital accumulation at the individual level. These externalities must be sufficiently strong, however, to generate the required result. We calculate a precise threshold value for the minimum magnitude of such externalities such that human capital accumulation becomes capable of driving aggregate growth if and only if this threshold is exceeded. We also put forward two alternative interpretations for these externalities, namely (i) pure knowledge spillovers [cf. Ben-Porath (1967)], and (ii) publicly provided physical capital in the human capital accumulation function. The latter has the interesting property that it relates to the question of private vs. public education funding [cf. Bénabou (1996)].[1]

The third contribution is that, taking advantage of our modeling approach, we find that the log-linear Mincerian relationship between wages (or human capital levels) and years of schooling cannot be carried forward from the micro to the macro scale due to insurmountable aggregation problems. *Even if the Mincerian relationship holds at the individual level, it is inevitably lost upon aggregation*, because of human capital depreciation due to births and deaths. This finding could explain why in empirical research, the Mincerian specification works much better at the micro level [e.g., Mincer (1974); Heckman et al. (2003)] than at the macro level of countries [e.g., Krueger and Lindahl (2001); Bloom et al. (2004)].

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The basic arguments of this paper are developed in Sections 2 and 3. In Section 2, the result that human capital accumulation cannot drive balanced growth if people's lives are finite and there are no human capital externalities is proven. It is also demonstrated why the Mincer equation does not hold at the macro level of countries after the vintage structure of human capital has been properly accounted for. In Section 3, it is shown how externalities in human capital accumulation (in the form of knowledge spillovers or public education spending) can rescue the balanced growth result. Section 4 concludes. Qualifications and robustness checks for our arguments have been relegated to the appendix.

2. HUMAN CAPITAL ACCUMULATION WITHOUT EXTERNALITIES

2.1. The Modeling Approach

The modeling approach that we are going to maintain throughout the paper abstracts from individual educational decisions of utility-maximizing individuals. Instead, we will presume that the division of time between schooling and working follows one of two simple rules of thumb: (i) that the time shares of these two activities are constant across time and age; (ii) that people first attend school full-time and then leave school and work full-time.

There are two reasons for proceeding this way. The first is that by disregarding the dynamic trade-offs inherent in endogenous choices of schooling effort, the impact of human capital aggregation on long-run growth and the earnings–schooling relationship is presented in a very transparent way: it is not blurred by the simultaneous incidence of other effects, unrelated to aggregation. The second is that our simplifying assumptions ensure analytical tractability of the model with externalities (the possibility of obtaining closed-form integrals, disentangling solutions from implicit equations, etc.). Certainly, only thanks to these assumptions are we able to obtain clear-cut predictions of the exact parametric conditions under which human capital accumulation drives or does not drive long-run growth. The extensions presented in the appendix are, however, reassuring that the main message conveyed herein is robust to a number of changes in the setup.

One unfortunate side effect of this approach is that if there is any dynamic interdependence between the aggregation of human capital in the society and individual educational choices, the current model cannot account for it.

2.2. The Model

Human capital embodied in each person is assumed to accumulate according to a linear differential equation. This equation is exactly the one that Lucas (1988) used in his aggregate specification: at each instant of time, the individual's human capital h is increased by the quantity $\dot{h} = (\lambda l_h + \mu l_Y)h$. The first component of the increment relates to the educational effort made by the individual ($\lambda > 0$ denotes the constant unit efficiency of education), whereas the second component relates to her work effort: hours worked increase work experience and thus labor productivity

($\mu \geq 0$ describes the pace at which experience is acquired).[2] Each individual is endowed with a fixed flow of time to divide between learning, working, and other activities (such as leisure or child rearing). We normalize this time endowment to unity and consequently impose the restriction that $l_h, l_Y \in [0, 1]$ as well as $l_h + l_Y \leq 1$.

More precisely, we write that each individual born at t, being currently at the age of τ, accumulates human capital according to

$$\frac{d}{d\tau} h(t, \tau) = [\lambda l_h(\tau) + \mu l_Y(\tau)] h(t, \tau). \tag{1}$$

Let us further assume that people are born with a constant initial level of human capital $h(t, 0) = h_0 > 0$ that does not depend on the time t at which the individual is born. The constancy of h_0 follows directly from its interpretation: h_0 is the natural level of useful abilities and skills, prior to all development. This is a rather innocuous, yet important assumption, because if for any reason (better nutrition, natural selection, genetic engineering, etc.), $h(t, 0)$ could grow over time, some of our results would be overturned.

The differential equation (1) can be solved for $h(t, \tau)$ yielding

$$h(t, \tau) = h_0 \exp \left[\lambda \int_0^\tau l_h(s) ds + \mu \int_0^\tau l_Y(s) ds \right]. \tag{2}$$

Assuming that there are no other factors in production than human capital, and that the production function exhibits constant returns to scale, it is easily inferred that the wage $w(t, \tau)$ is equal to individual human capital. We thus obtain a variant of the Mincerian wage equation where log wages are a linear function of total schooling effort and cumulative work experience:

$$w(t, \tau) = h(t, \tau) = h_0 \exp \left[\lambda \int_0^\tau l_h(s) ds + \mu \int_0^\tau l_Y(s) ds \right]. \tag{3}$$

Furthermore, the Mincerian wage equation is preserved even if there are other factors in production, such as physical capital or unskilled labor, provided that the production function is Cobb–Douglas: log wages would then not be equal, but still be *proportional* to log human capital.

This reasoning leads to the following proposition.

PROPOSITION 1. *If each individual of age τ first engages in full-time education ($l_h = 1$) for time τ_h and then works full-time ($l_Y = 1$) for time τ_Y, with $\tau_h + \tau_Y \leq \tau$, then*

$$h = h_0 \exp (\lambda \tau_h + \mu \tau_Y) \quad \Leftrightarrow \quad \ln h = \ln h_0 + \lambda \tau_h + \mu \tau_Y. \tag{4}$$

If wages are proportional to human capital, then equation (4) is directly the Mincerian wage equation [Mincer (1974)].

Proof. Insert appropriate formulas for $l_h(s)$ and $l_Y(s)$ into equation (2) and compute the resulting integrals. ∎

The above proposition states that our model predicts log wages to be linear in years of schooling and years of work experience.[3] This is a *micro-level* relationship, because it refers to wages, years of schooling, and years of work experience as measured for a given individual at a single moment in time.

As they stand, equations (2)–(4) apply only to individuals born at the same time t. They would also hold for a cross section of people born at different times, however, provided that the efficiency parameters λ and μ were constant over time.

2.3. Aggregation across Individuals

Let us now turn to the demographics. To keep the basic model as simple as possible, we will assume that at each moment in time $t \in (-\infty, +\infty)$, there exist a continuum of people of measure $N(t)$. We will also suppose that the birth rate is constant, age-invariant, and equal to $b > 0$—that is, the number of births at each moment in time t is proportional to the total population and equal to $bN(t)$. Furthermore, we will tentatively limit ourselves to the simplest "perpetual youth" case, implying that the hazard rate of death faced by each individual is constant, independent of age, and equal to $d > 0$ [cf. Blanchard, (1985)]. This means that the unconditional probability of surviving to an age of τ and the conditional probability of surviving an additional τ years are equal and decreasing exponentially: $m(\tau) = e^{-d\tau}$.[4] Given these simplifying assumptions and the Law of Large Numbers, it follows that the population growth rate is deterministic, constant, and equal to $\hat{N}(t) = b - d$ for all t. If $N(0) = N_0$ then there are $N(t) = N_0 e^{(b-d)t}$ people alive at t.

The age structure of the population alive at t is easy to compute. Naturally, to be τ years old at t, one has to (i) be born at $t - \tau$, and (ii) survive to the age of τ. This implies that there are $P(t, \tau)$ people aged τ in the population, with

$$P(t, \tau) = bN(t - \tau)m(\tau) = bN_0 e^{(b-d)(t-\tau)} e^{-d\tau} = N(t) b e^{-b\tau}. \qquad \textbf{(5)}$$

The average level of human capital in the society is then given by[5]

$$\bar{h}(t) = \int_0^\infty \frac{P(t, \tau) h(t - \tau, \tau)}{N(t)} d\tau = b \int_0^\infty h(t - \tau, \tau) e^{-b\tau} d\tau. \qquad \textbf{(6)}$$

If individuals spend constant percentages of their time endowment on learning and working, $\bar{l}_h$ and $\bar{l}_Y$ respectively, we get

$$\bar{h}(t) = b \int_0^\infty h_0 e^{(\lambda \bar{l}_h + \mu \bar{l}_Y)\tau} e^{-b\tau} d\tau = \frac{b h_0}{b - \lambda \bar{l}_h - \mu \bar{l}_Y}, \qquad \textbf{(7)}$$

provided that $b > \lambda \bar{l}_h + \mu \bar{l}_Y$, so that the few arbitrarily old people with arbitrarily high human capital levels (existence of such individuals is an unrealistic implication of the "perpetual youth" survival law that does not impose any upper bound on people's lifespans) do not dominate the population, causing the average human capital level to diverge.[6]

All comparative statics following from equation (7) are standard: $\bar{h}(t)$ increases with λ, μ, $\bar{l}_h$, $\bar{l}_Y$, h_0 and decreases with b. The derived *hyperbolic* pattern of dependence of $\bar{h}(t)$ on the efficiency parameters [with $\bar{h}(t) \to \infty$ as $\lambda\bar{l}_h + \mu\bar{l}_Y \to b$] is, however, not a robust finding but rather an artifact of the assumed "perpetual youth" survival law. Nevertheless, aggregate human capital will typically *not* follow the Mincerian pattern of dependence on years of schooling and work experience even if such a pattern is found in individual data:

PROPOSITION 2. *Under a stationary age structure, the relationship between aggregate human capital $\bar{h}(t)$ and average years of schooling, proportional to $\lambda\bar{l}_h$, is not log-linear unless the survival function m depends on years of schooling in one crucial (and arguably implausible) way.*

Proof. See the working paper version of this article, Growiec (2007). ∎

The finding that aggregate Mincer equations are incompatible with intergenerational aggregation calls into question the validity of the popular practice of carrying the cross-sectional Mincerian relationship forward to dynamic growth models with a representative agent [see, e.g., Jones (2005)]. Our analysis implies that the static equation $h = e^{\psi l_h}$, where h is the average human capital level in the society and l_h is the average time share of schooling, cannot be reproduced under finite lifetimes, at least in the absence of human capital externalities.

Another general observation is that because each individual's human capital depends on her age but not on the time at which she was born, that is, $h(t, \tau)$ does not depend on t, it follows automatically that under a stationary age structure (which allows the population itself to be exponentially growing or declining; see Appendix A.6), the average level of human capital in the society $\bar{h}(t)$ will not depend on t either, independent of the presumed survival law. Hence, it will be constant over time, just as in the "perpetual youth" case discussed above:

PROPOSITION 3. *If the age structure of the population is stationary, then the average level of human capital in the society $\bar{h}(t)$ is constant over time.*

Proof. Already given in text. ∎

This result should be contrasted with the human capital–based macro growth literature that assumes aggregate human capital to grow over time, e.g., Uzawa (1965), Lucas (1988, 1993), Barro and Sala-i-Martin (1995, Chapter 5), Gong et al. (2004), and numerous other articles in this vein. It turns out that under finite lifetimes, and in the absence of externalities, human capital accumulation cannot work as an engine of aggregate growth even if the production function at the individual level is linear.

3. HUMAN CAPITAL EXTERNALITIES

It is not true that there is no force able to generate balanced growth in the simple model of the preceding section. In fact, growth can arise there because of the following:

Unit efficiencies of schooling and work effort $\lambda(t)$ and $\mu(t)$ may increase over time [cf. Arias and McMahon (2001)].
Human capital at birth $h(t, 0)$ may increase over time.[7]
The demographic structure (age profile) of the society may change over time.[8]

In all three cases, however, the human capital accumulation sector can only help enhance, or facilitate, economic growth, but the fundamental "growth engine" is in technological progress or demographics. For this reason, we will now set these options aside and concentrate on human capital externalities, which offer a unique possibility of long-run growth driven by human capital accumulation alone.

3.1. A Case for Human Capital Externalities

But are there human capital externalities in reality? It seems so, because it is often argued in the empirical literature that there exist substantial social returns to education on top of the individually appropriable private returns. Unfortunately, social returns to schooling are notoriously difficult to estimate.[9] For example, in a study by Acemoglu and Angrist (2001), social returns to schooling are found to be negligibly small. Most studies, however, find positive and significant social returns. Davies (2003) concludes his review of this literature with a statement that "it is possible that education externalities could amount to something like 6–8% points."

This said, let us discuss the potential sources of social returns to schooling. The first, most natural source would be pure knowledge spillovers, appearing at the level of family and local community (e.g., school district) as well as the whole society [cf. Bénabou (1996); Tamura (2001); Rangazas (2005)].

The second potential source of human capital externalities is related to public education spending. If the human capital accumulation technology requires physical capital inputs, then public education spending can create externalities because physical capital will be then provided in proportion to the total (or average) human capital in the population. Private education spending, on the other hand, being a function of one's own human capital, cannot play such a role—at least, unless it assumes some form of resource pooling.

A simple setup suitable for incorporating physical capital in the human capital accumulation technology will be discussed in Section 3.3. We shall confirm that public spending is capable of producing results akin to knowledge spillovers and that in priniciple, private education spending leaves the results of the no-externalities model unchanged.[10]

3.2. Introducing Externalities into the Model

Human capital externalities will be introduced into our basic model by assuming that the increments to individual human capital are proportional not just to one's own human capital, but to a CRTS Cobb–Douglas bundle of one's own human capital $h(t,\tau)$ and the average human capital in the society $\bar{h}(t)$ [cf. Ben-Porath (1967); Rangazas (2000); Tamura (2001)].[11] This crucial modification will change the results because, as opposed to individual human capital, average human capital is not embodied in any particular person; it may increase due to schooling and on-the-job training, and decrease due to births and deaths, but its overall evolution can go in either direction, depending on the relative strength of the forces at play.

Disregarding on-the-job training for simplicity ($\mu = 0$), we obtain the following human capital accumulation equation at the individual level:

$$\frac{d}{d\tau}h(t,\tau) = \lambda l_h(\tau)[h(t,\tau)]^{\theta}[\bar{h}(t+\tau)]^{1-\theta}, \qquad \theta \in [0,1]. \tag{8}$$

The parameter θ captures the relative share of one's own human capital in human capital accumulation. $\theta = 1$ captures the no-externalities case of Section 2. In the second polar case, $\theta = 0$, the share of individual human capital in its own accumulation is nil: education is then exclusively about transferring knowledge from teachers to pupils and not at all about individual learning. We shall see shortly that the case $\theta = 0$ offers especially transparent results.

Coupled with the initial condition $h(t,0) = h_0 > 0$, equation (8) is solved as follows:

$$h(t,\tau) = \left\{(1-\theta)\lambda \int_0^{\tau} l_h(s)[\bar{h}(t+s)]^{1-\theta} ds + h_0^{1-\theta}\right\}^{\frac{1}{1-\theta}}. \tag{9}$$

The first observation is that the cross-sectional relationship between individuals' human capital and their cumulative learning effort is no longer log-linear (Mincerian) if there are externalities in schooling.

Aggregating human capital accross generations as in equation (6), using the equality (9) and the "perpetual youth" survival law, we obtain the following integral equation, which implicitly defines $\bar{h}(t)$:

$$\bar{h}(t) = b\int_0^{\infty} e^{-b\tau}\left\{(1-\theta)\lambda \int_0^{\tau} l_h(s)[\bar{h}(t-\tau+s)]^{1-\theta} ds + h_0^{1-\theta}\right\}^{\frac{1}{1-\theta}} d\tau. \tag{10}$$

To check whether this formulation can give rise to balanced growth in $\bar{h}(t)$, we proceed as follows. We insert an exponential solution $\bar{h}(t) = \bar{h}_0 e^{\beta t}$ into (10) and calculate the integrals under the assumption that $l_h = \bar{l}_h \equiv \text{const}$ (which we make for tractability). We then specify the conditions under which $\beta > 0$.

Because our objective is only to verify whether the model with externalities is consistent with balanced growth, the following analysis will be limited to real values of β. The reason is that only for real β's can the temporal evolution of $\bar{h}(t)$ follow an exponential growth pattern; for complex β's with a nonzero imaginary part, equation (10) would imply oscillatory behavior in which we are not interested here. It must be noted, however, that the characteristic equation of (10) is a transcendental equation, having an infinity of complex roots, so that transitional dynamics and stability are quite difficult questions.

It turns out that in certain cases, the model is consistent with balanced growth, but this result depends crucially on the magnitude of externalities, as measured by $1-\theta$, and on the relative efficiency of education $\lambda\bar{l}_h$ as compared to the birth rate b. Working with the limit $t \to \infty$ under the restriction that β be real, we obtain the following proposition.

PROPOSITION 4. *Assume $\theta \in [0, 1)$ and $\bar{l}_h \equiv$ const. There exists a unique balanced growth path that has aggregate human capital $\bar{h}(t)$ growing exponentially over time with a growth rate $\beta > 0$ if and only if*

$$\lambda\bar{l}_h > \frac{b}{(1-\theta)\left[\Gamma\left(\frac{2-\theta}{1-\theta}\right)\right]^{1-\theta}}. \qquad \textbf{(11)}$$

If the sign in (11) is reversed, then there exists a unique steady state $\bar{h}_0$ where aggregate human capital is constant over time.[12]

Proof. Insert an exponential solution $\bar{h}(t) = \bar{h}_0 e^{\beta t}$ into (10). It is obtained that

$$\bar{h}_0 e^{\beta t} = b\int_0^\infty e^{-b\tau}\left[\frac{\lambda l_h}{\beta}\bar{h}_0^{1-\theta}e^{-\beta(1-\theta)t}\left(1-e^{-\beta(1-\theta)\tau}\right)+h_0^{1-\theta}\right]^{\frac{1}{1-\theta}} d\tau. \qquad \textbf{(12)}$$

Case $\beta > 0$. In such case, we can divide (12) sidewise by $\bar{h}_0 e^{\beta t}$ and take the limit $t \to \infty$ so that the term with h_0 disappears. The following characteristic equation is obtained:

$$\Psi(\beta) = 1 - b(\lambda\bar{l}_h)^{\frac{1}{1-\theta}}\int_0^\infty e^{-b\tau}\left(\frac{1-e^{-\beta(1-\theta)\tau}}{\beta}\right)^{\frac{1}{1-\theta}} d\tau = 0. \qquad \textbf{(13)}$$

We will show that $\Psi(\beta)$ crosses zero exactly once in its domain $\beta > 0$ (and thus a unique growth rate β exists) if and only if (11) is satisfied. First, note that $\lim_{\beta\to\infty}\Psi(\beta) = 1 > 0$. Second, calculate the derivative $\Psi'(\beta)$:

$$\Psi'(\beta) = -b(\lambda\bar{l}_h)^{\frac{1}{1-\theta}} \qquad \textbf{(14)}$$
$$\times\int_0^\infty e^{-b\tau}\frac{1}{1-\theta}\left(\frac{1-e^{-\beta(1-\theta)\tau}}{\beta}\right)^{\frac{\theta}{1-\theta}}\frac{e^{-\beta(1-\theta)\tau}[1+\beta(1-\theta)\tau]-1}{\beta^2}d\tau.$$

All factors of the derivative can are unambiguously positive but for the minus in front of the expression and the factor $e^{-\beta(1-\theta)\tau}[1+\beta(1-\theta)\tau]-1$, which is negative, because $e^x > 1+x$ for all $x \neq 0$. We can thus conclude that $\Psi'(\beta) > 0$, so Ψ is increasing in its whole positive domain. Hence, a unique $\beta > 0$ such that $\Psi(\beta) = 0$ exists if and only if $\lim_{\beta\to 0_+}\Psi(\beta) < 0$. From l'Hôpital's rule, we obtain $\lim_{\beta\to 0_+}\frac{1-e^{-\beta(1-\theta)\tau}}{\beta} = (1-\theta)\tau$. Finally,

$$\lim_{\beta\to 0_+}\Psi(\beta) = 1 - b[(1-\theta)\lambda\bar{l}_h]^{\frac{1}{1-\theta}}\int_0^\infty e^{-b\tau}\tau^{\frac{1}{1-\theta}}d\tau \qquad (15)$$

$$= 1 - \left[\frac{(1-\theta)\lambda\bar{l}_h}{b}\right]^{\frac{1}{1-\theta}}\Gamma\left(\frac{2-\theta}{1-\theta}\right).$$

Rearranging this slightly, we get $\lim_{\beta\to 0_+}\Psi(\beta) < 0 \Leftrightarrow \lambda\bar{l}_h > b/(1-\theta)[\Gamma(\frac{2-\theta}{1-\theta})]^{1-\theta}$.

Case $\beta < 0$. In this case, taking the limits as $t \to \infty$ on both sides of (12) yields $0 = b\int_0^\infty e^{-b\tau}h_0 d\tau = h_0$, which contradicts the assumption $h_0 > 0$. This case is thus impossible.

Case $\beta = 0$. Inserting $\bar{h}(t) \equiv \bar{h}_0$ into (10) and dividing sideways by $\bar{h}_0$, we obtain

$$\Xi(\bar{h}_0) = b\int_0^\infty e^{-b\tau}\left[(1-\theta)\lambda\bar{l}_h\tau + \left(\frac{h_0}{\bar{h}_0}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}d\tau - 1 = 0. \qquad (16)$$

We will now show that a unique solution $\bar{h}_0 \geq h_0$ to this equation exists if and only if $\lambda\bar{l}_h < b/(1-\theta)[\Gamma(\frac{2-\theta}{1-\theta})]^{1-\theta}$. First, it is easy to see that

$$\Xi(h_0) = b\int_0^\infty e^{-b\tau}[(1-\theta)\lambda\bar{l}_h\tau + 1]^{\frac{1}{1-\theta}}d\tau - 1 > b\int_0^\infty e^{-b\tau}d\tau - 1 = 0. \qquad (17)$$

Second, we calculate the derivative of Ξ:

$$\Xi'(\bar{h}_0) = -b\int_0^\infty e^{-b\tau}\left[(1-\theta)\lambda\bar{l}_h\tau + \left(\frac{h_0}{\bar{h}_0}\right)^{1-\theta}\right]^{\frac{\theta}{1-\theta}}d\tau\cdot\frac{h_0^{1-\theta}}{\bar{h}_0^{2-\theta}} < 0. \qquad (18)$$

Thus, there exists a unique $\bar{h}_0$ such that $\Xi(\bar{h}_0) = 0$ if and only if $\lim_{\bar{h}_0\to\infty}\Xi(\bar{h}_0) < 0$. Analogously to the calculations above, we obtain

$$\lim_{\bar{h}_0\to\infty}\Xi(\bar{h}_0) = \left[\frac{(1-\theta)\lambda\bar{l}_h}{b}\right]^{\frac{1}{1-\theta}}\Gamma\left(\frac{2-\theta}{1-\theta}\right) - 1. \qquad (19)$$

Rearranging this expression, we find that $\lim_{\bar{h}_0\to\infty}\Xi(\bar{h}_0)$ is negative if and only if $\lambda\bar{l}_h < b/(1-\theta)[\Gamma(\frac{2-\theta}{1-\theta})]^{1-\theta}$, which completes the proof. ■

We note that the characteristic equation (13) reduces to $\beta = \lambda\bar{l}_h - b$ for $\theta = 0$: the unique growth rate β is positive if $\lambda\bar{l}_h > b$—i.e., if the pace of learning outruns the flow of births [people are born uneducated and the role of deaths is nil in the perpetual youth case; cf. Faruqee (2003)]. If the flow of births diluting average human capital is greater than the pace of human capital accumulation through schooling, aggregate human capital will stagnate.

Furthermore, the long-run growth rate is uniformly lower than the instantaneous rate of human capital formation through schooling ($\lambda\bar{l}_h$), because of human capital depreciation due to births and deaths:

PROPOSITION 5. *The balanced growth rate β is always lower than $\lambda\bar{l}_h$.*

Proof. If $\beta = 0$ then trivially $\beta < \lambda\bar{l}_h$. Assume in turn that $\beta > 0$. In this case, from (14) we know that $\Psi'(\beta) > 0$. To show that $\Psi(\beta) = 0$ for $\beta < \lambda\bar{l}_h$, it suffices then to show $\Psi(\lambda\bar{l}_h) > 0$. This inequality holds because

$$\Psi(\lambda\bar{l}_h) = 1 - b\int_0^{\infty} e^{-b\tau}\left(1 - e^{-\beta(1-\theta)\tau}\right)^{\frac{1}{1-\theta}} d\tau > 1 - b\int_0^{\infty} e^{-b\tau}\, d\tau = 0. \qquad \blacksquare$$

We have found that assuming strong society-level spillovers $1 - \theta$ or a high schooling intensity $\lambda\bar{l}_h$ as required by (11) can rescue the balanced growth result and—in a sense—help construct a "vintage Uzawa–Lucas model" where aggregate human capital is accumulated according to a linear technology and is subject to depreciation with a constant rate b. In effect, our microfounded aggregate human capital accumulation equation resembles, at least in the BGP approximation,[13] the Uzawa–Lucas equation $\dot{\bar{h}}(t) = \beta\bar{h}(t)$, which gives rise to long-run balanced growth as long as $\beta > 0$. Furthermore, if $\theta = 0$, then $\beta = \lambda\bar{l}_h - b$, and thus the growth rate is directly the difference between the unit efficiency of education and the birth rate.

While the case $\theta = 0$ is clearly implausible [Rangazas (2000); Tamura (2001); Manuelli and Seshadri (2005)], there exists substantial empirical evidence that θ actually falls in the range (0.75, 0.8): see Borjas (1995) and Rangazas (2000). If $\theta = 0.75$, then for balanced growth it is required—according to equation (11)—that $\lambda\bar{l}_h > 1.8072b$; if $\theta = 0.8$, the condition becomes $\lambda\bar{l}_h > 1.9193b$.

If equation (11) does *not* hold, then the role of human capital at birth (h_0) is not negligible in the long run. A unique steady state $\bar{h}_0$ exists then such that $\bar{h}_0$ depends on h_0. We also find that $h_0 > 0$ imposes a lower bound on the aggregate human capital level. Exponential decline in $\bar{h}(t)$ is thus ruled out even though the characteristic equation (13) has a negative root.

Another observation is that the lower bound imposed on schooling intensity by the balanced growth requirement [the right-hand side of (11)] depends on the birth rate b positively and linearly, so that the greater the birth rate, the less viable is long-run growth driven by aggregate human capital accumulation. We also find its positive dependence on θ—i.e., negative dependence on the magnitude of knowledge spillovers. The factor multiplying b on the right-hand side of (11) rises smoothly from 1 when $\theta = 0$ to e when $\theta \to 1$. This result is also intuitive: the stronger are spillovers in education, the greater are the opportunities for balanced growth (and the faster is growth, as we shall prove below).

Apart from the special case $\theta = 0$ where $\beta = \lambda \bar{l}_h - b$, the growth rate of aggregate human capital cannot be explicitly calculated: the characteristic equation (13) does not offer explicit formulas. However, we can still infer the direction of dependence between β and crucial parameters of the model. The following proposition holds.

PROPOSITION 6. *The asymptotic balanced growth rate β depends positively on effective schooling intensity $\lambda \bar{l}_h$ and negatively on the birth rate b and the share of one's own human capital in the schooling technology, θ.*

Analogous calculations may be carried out for the level of aggregate human capital in the asymptotic steady state $\bar{h}_0$ if balanced growth is ruled out. We obtain the following results:

PROPOSITION 7. *The level of aggregate human capital in the asymptotic steady state $\bar{h}_0$ depends positively on effective schooling intensity $\lambda \bar{l}_h$ and human capital at birth h_0, but negatively on the birth rate b and the share of one's own human capital in the schooling technology, θ.*

All results summarized in the propositions above conform with our initial intuitions: greater schooling intensity $\lambda \bar{l}_h$ has an unambiguously positive impact either on the long-run growth rate of the economy (if growth is feasible) or on the steady-state human capital level (if growth is not feasible). This positive impact, present already in the short run, carries forward to the long run without any disturbances. The same reasoning can be applied to the birth rate b, responsible for the depreciation of aggregate human capital. Furthermore, stronger spillovers imply more growth (or a higher steady-state level of human capital) because they reduce the role of replacement investment: although individual human capital is embodied and thus lost upon death, aggregate human capital has the disembodied character and "lives on" despite deaths and births.

3.3. Introducing Physical Capital: Public versus Private Education Spending

We will now discuss the other possibility for generating externalities in human capital accumulation, that is, via public education spending. To this end, we will allow physical capital to be a factor in the human capital accumulation technology [cf. Rebelo (1991)]. For simplicity, we will retain the linear production function for consumption goods, implying $w(t, \tau) = h(t, \tau)$,[14] and assume a constant rate of education spending.[15]

Let us first consider the case of private education spending. Abstracting from bequests and parental funding, and assuming that the rate of spending on education out of wages $s \in (0, 1)$ is constant across time and ages of the individuals, we get that the physical capital input in the schooling technology is equal to

$$k(t,\tau) = sw(t,\tau) = sh(t,\tau). \tag{20}$$

Replacing the human capital accumulation equation from the baseline model (1) with one taking a CRTS Cobb–Douglas bundle of physical and human capital as its input, we get

$$\frac{d}{d\tau}h(t,\tau) = (\lambda\bar{l}_h + \mu\bar{l}_Y)h^{\alpha}(t,\tau)k^{1-\alpha}(t,\tau) = (\lambda\bar{l}_h + \mu\bar{l}_Y)s^{1-\alpha}h(t,\tau), \tag{21}$$

which implies that, qualitatively, the current case is equivalent to our baseline model with no human capital externalities. Average human capital $\bar{h}(t)$ is constant across time, implying that human capital accumulation cannot be an engine of growth; the Mincerian equation holds at the individual level but does not hold in the aggregate.

It should be noted that allowing intergenerational bequests (a percentage of wage transferred from parents to children) would not overturn these results (cf. Appendix A.5). These results would be overturned, however, if private education spending pooled resources from some heterogeneous group of people, such as a vertical section of the society. It would then act just like public education spending, discussed below.

In the case of public education spending, public provision of capital goods can introduce an externality from aggregate human capital, providing us with an important argument that the magnitude of these spillovers, $1-\theta$ in equation (8), could be reasonably high in reality. Assuming that the government levies a personal income tax at a fixed rate T, we obtain that effective wages are equal to $(1-T)h(t,\tau)$, whereas the total tax revenue collected by the state (and immediately spent on public education) is

$$\bar{T}(t) = \int_0^{\infty} P(t,\tau)Th(t-\tau,\tau)d\tau = TN(t)\bar{h}(t). \tag{22}$$

Assuming an equal division of all collected taxes among all individuals alive at t, the physical capital input to human capital accumulation is equal to

$$\bar{k}(t) = \frac{\bar{T}(t)}{N(t)} = T\bar{h}(t). \tag{23}$$

Hence, the human capital accumulation equation becomes

$$\frac{d}{d\tau}h(t,\tau) = (\lambda\bar{l}_h + \mu\bar{l}_Y)h^{\alpha}(t,\tau)\bar{k}^{1-\alpha}(t) = (\lambda\bar{l}_h + \mu\bar{l}_Y)T^{1-\alpha}h^{\alpha}(t,\tau)\bar{h}(t)^{1-\alpha}. \tag{24}$$

In qualitative terms, the human capital accumulation equation is thus now equivalent to the one featuring pure knowledge spillovers, summarized in equation (8), with $\theta = \alpha$.

In sum, from the two simple setups discussed above, we learn that public education spending can constitute an important source of human capital externalities, markedly increasing the chance that human capital accumulation will drive growth, whereas private education spending cannot have such effects, even if parental bequests are allowed for.

A question remains as to whether such public education externalities are empirically plausible. The answer is unfortunately difficult to judge because of the admittedly simplified form of the considered model. One has to keep in mind that in reality, public spending might be associated with low spending rates, and private spending could well be partially "public" in the sense of the distinction made in the above theory: it might pool resources over a group of people with different levels of human capital and thus earnings [e.g., a school district; cf. Bénabou (1996); Tamura (2001)].

3.4. A Mixed Case

Let us finally consider a mixed case where education spending is provided both privately and publicly, and where there exist pure knowledge spillovers on top of the externalities obtained thanks to the public provision of physical capital. In this case, one's own human capital $h(t,\tau)$ and the spillover term $\bar{h}(t)$ would enter the human capital production function in the following way:

$$\frac{d}{d\tau}h(t,\tau) = (\lambda\bar{l}_h + \mu\bar{l}_Y)(s^{\zeta}T^{1-\zeta})^{1-\alpha}h(t,\tau)^{\alpha\theta+(1-\alpha)\zeta}\bar{h}(t)^{\alpha(1-\theta)+(1-\alpha)(1-\zeta)}, \tag{25}$$

where α captures the total share of human capital in the education technology, θ captures the share of one's own human capital in the total human capital bundle, and ζ captures the percentage share of private education spending. The original elasticity of the spillover term, $1-\theta$, is now replaced by

$$1-\xi = \alpha(1-\theta) + (1-\alpha)(1-\zeta), \tag{26}$$

which reduces to $1-\theta$ if $\alpha = 0$ (no physical capital) and to $1-\alpha$ if $\theta = 1$ and $\zeta = 0$ (no knowledge spillovers, purely public education spending).

3.5. Relation to the Uzawa–Lucas Model

With finite lifetimes, and in the absence of human capital externalities, human capital accumulation cannot give rise to endogenous balanced economic growth along the lines of the Uzawa–Lucas model [Uzawa (1965); Lucas (1988; 1993); Barro and Sala-i-Martin (1995, Chapter 5); Gong et al. (2004)], even if the human capital accumulation equation is linear at the individual level. The reason is that by assuming the representative agent to be infinitely lived (or under an alternative interpretation, by assuming human capital to be disembodied), the model ignores the depreciation of aggregate human capital due to deaths of the human-capital-rich and births of the human-capital-poor.

The assumptions of the Uzawa–Lucas model may be rescued, however, thanks to human capital externalities, but only under the condition that inequality (11) holds, i.e., that schooling intensity $\lambda \bar{l}_h$ is high enough to outweigh human capital depreciation. The crux of the argument is that the linear production function of form $\dot{\bar{h}} = B\bar{h}$, where $B > 0$, postulated in the Uzawa–Lucas model, can be interpreted as $\dot{\bar{h}} = A\bar{h} - \delta\bar{h}$, where $A > \delta$ and δ is the human capital depreciation rate, which must be positive, due to births and deaths. Such a linear production function at the aggregate level requires that (i) the production function at the individual level is also linear, and (ii) human capital externalities are strong enough. Otherwise, the marginal product of human capital would gradually fall to zero instead of remaining constant.

3.6. Relation to Jones and Manuelli (1992)

Jones and Manuelli (1992) have analyzed the impact of finite lifetimes on the ability of discrete-time overlapping-generations models to generate balanced growth. Their work is related to ours in the following way. First, Jones and Manuelli have demonstrated how the impossibility of passing capital (predominantly physical capital) across generations can preclude balanced growth, even if the production function in the economy is such that the marginal product of capital is bounded away from zero (for example, if it is of the AK type). We have essentially reproduced their result with human capital (and in continuous time). Second, they have explained how a public policy consisting of redistributing wealth from the old to the young can rescue balanced growth; we have concluded that with respect to human capital, an equivalent result would require strong enough human capital externalities.

There is one noteworthy difference between the two contributions (apart from the different modeling approaches): human capital, unlike physical capital, is embodied in people and cannot be transferred directly across generations. The case considered here is thus much more serious: there exists a firm natural constraint that precludes direct transfers of human capital across generations, which is not the case with physical capital. In effect, Jones and Manuelli's limits to growth could, for example, be overcome by introducing intergenerational altruism, and ours could not.

4. CONCLUSIONS

The fact that human capital is embodied in people whose lifespans are finite has far-reaching consequences both for economic growth theory and for the associated empirical literature, two of which have not been acknowledged yet:

(i) human capital accumulation cannot drive aggregate growth unless there are strong enough human capital externalities (in the form of pure knowledge spillovers or public education spending);
(ii) the age structure of the society has an impact on the pattern of dependence between aggregate human capital and average years of schooling. The log-linear (Mincerian) relationship between these two variables is inevitably lost upon aggregation.

The contribution of this paper to the literature is to prove the above claims. To do so, we have carried out the procedure of aggregating human capital across individuals, taking into account the explicit vintage structure of the society, and the fact that differences in age imply heterogeneity in human capital levels.

One of the most important results of this paper is a precise threshold value for the minimum strength of human capital externalities required for human capital accumulation to drive aggregate growth. We have also demonstrated why, and to which extent, public education spending could contribute to these externalities alongside knowledge spillovers.

A suggestion for further theoretical work would be to investigate the consequences of dynamic changes in the demographic structure [cf. Boucekkine et al. (2002)] for the aggregate human capital level. Another idea would be to allow survival laws to depend positively on the level of individual human capital. This could alter the results contained herein. Finally, one could try to assess the roles of technological progress, human capital accumulation, and technology–skill complementarity in generating growth under embodied human capital with an explicit vintage structure.

As a suggestion for further empirical research, we would hint at paying special attention to the demographic structure of the population whose human capital is aggregated. Because, due to aggregation problems, the Mincer equation does not hold at the macro level of countries, another question for further inquiry [already addressed by, e.g., Bils and Klenow (2000)] is what is the accurate functional form for the aggregate wages–schooling relationship.

Above all, however, three simple facts should never be disregarded in human capital theory: (i) that human capital is embodied in people, (ii) that people differ in age, and (iii) that in the end, each of us is going to die.

NOTES

1. It is also argued that human capital speeds the adoption of new technologies and is strongly complementary to technology [Bils and Klenow (2000)]. However, if technology adoption were the main channel of impact of human capital on growth, then the source of growth would be not human capital accumulation itself but technological progress. We shall thus ignore this possibility.

2. A similar derivation has been put forward by Mincer (1974) and later discussed by Heckman et al. (2003). The difference is that we present our version of Mincer's discrete-time equation $h_{t+1} = (1 + \rho_t)h_t$ as a restrictive assumption on the schooling technology, whereas in those two works it is presented as an *accounting identity*. However, these authors immediately assume that ρ_t—the rate of return on formal schooling—is constant over all years of schooling. Even if one indeed thought of the individual's human capital accumulation equation as an accounting identity, this constancy would be the key restriction. It does not follow from the identity but is imposed arbitrarily.

3. The difference between this result and Mincer (1974) is that we omit the squared term in work experience. This is because, for simplicity, we have abstracted from the finding that on-the-job training is characterized not by constant but by decreasing returns. See equation (1) in Krueger and Lindahl (2001).

4. Using more realistic survival laws, implying that youth is "fleeting" rather than "perpetual," i.e., Boucekkine et al. (2002, 2003) or Faruqee (2003), does not contradict the main message conveyed herein. See the appendix.

5. Implicit in our aggregation exercise is the assumption that skill levels are perfectly substitutable. Pandey (2008) argues, however, that the actual elasticity of substitution between skilled and unskilled labor is not infinite, but around 4, when estimated using worldwide cross-country data. We leave this problem for further work.

6. Manuelli and Seshadri (2005) carry out a similar aggregation exercise for their human capital–based general equilibrium model.

7. This could be the case if there existed "societal human capital, that is, knowledge freely available to individuals that they inherit simply because they are born" [Jones and Manuelli (1992)].

8. Ongoing increases in longevity and decreases in the birth rate have been shown to raise the average human capital level and thus render more growth possible, at least temporarily [cf. Boucekkine et al. (2002, 2003); Azomahou et al. (in press)].

9. Standard estimates of the magnitude of *private* returns to an additional year of schooling fall in the range between 6% and 10% [Card (1999)]. Arias and McMahon (2001) argue that these estimates, based on cross-sectional studies, may be biased downwards due to the persistent upward trend in average earnings. Belzil and Hansen (2002) argue that "contrary to conventional wisdom, the log wage regression is . . . convex in schooling."

10. To some extent, this parallels Bénabou (1996), who has analyzed a general reduced form of a human capital accumulation equation in a discrete-time overlapping-generations setup. His focus was, however, on intracohort heterogeneity and not vintage effects. Furthermore, Bénabou implicitly assumed that parental human capital entered the human capital accumulation function. He has thus introduced knowledge spillovers capable of driving growth alone (see Appendix A.1) to his model, on top of the discussion of public vs. private education spending (and segregation vs. integration).

11. Average human capital $\bar{h}(t)$ entering the externality term does not have to be averaged across the whole society: if there was intracohort heterogeneity, it could also make sense to consider more localized externalities [cf. Bénabou (1996)]. Furthermore, in discrete-time overlapping-generations models, there exists a concealed way of introducing such externalities without acknowledging them: that is, taking parents' human capital as input in the human capital production function. For a discussion of why this is equivalent to introducing externalities from average human capital, please consult Appendix A.1.

12. The symbol Γ refers to Euler's Gamma function, $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$.

13. Stability properties of the two models will most likely differ.

14. Assuming that physical capital enters the production function for consumption goods in a Cobb–Douglas manner would not alter any of our predictions [cf. Rebelo (1991); Barro and Sala-i-Martin (1995, Chapter 5)].

15. Relaxing this assumption would make it necessary to deal with complex optimization problems, whose results could render aggregation of human capital across generations analytically intractable and blur our clear-cut results by introducing additional trade-offs.

16. The results can easily be extended to models where individuals live for $n = 1, 2, \ldots$ periods.

17. In papers dealing with intracohort heterogeneity, such as de la Croix and Doepke (2003), a distinction must be made between direct *parental* human capital h_{it}^O and the average human capital in the parents' generation $\bar{h}_t^O$. In such case, endogenous inequality may occur, but long-run growth in *average* human capital will follow only if the combined contribution of h_{it}^O and $\bar{h}_t^O$ to $h_{i,t+1}^O$ is strong enough.

18. This assumption represents the idea that teaching at elementary school does not require the knowledge necessary for lecturing at a university. The knowledge effectively transferred to a pupil is

more closely related to the kid's current grade at school than to the average human capital level in the society.

19. It can be shown that $\omega > 0$ and $m > 1$ are related by the implicit equation $\omega(m\lambda\bar{l}_h + \mu\bar{l}_Y) - \ln m = 0$. There exist two positive solutions $m_1(\omega)$ and $m_2(\omega)$ if $\omega\mu\bar{l}_Y + \ln\omega + \ln(\lambda\bar{l}_h) + 1 < 0$ and one positive solution if it is equal to zero.

20. For any survival law, equation (A.13) is automatically satisfied with $d^* = b$: by the sole fact that m is a survival law, i.e., $1 - m$ is a cumulative distribution function concentrated on $[0, +\infty)$, it follows that $\int_0^\infty m'(\tau)d\tau = -1$. This trivial solution is, however, introduced by differentiation of both sides of the equation $N(t) = \int_{-\infty}^t bN(s)m(t-s)ds = N_0 e^{(b-d)t}$ with respect to time t. It does not carry any substantial economic meaning.

21. The proof of this result uses the fact that the left-hand side of (A.13) is continuous and concave in d and that it is equal to zero for $b = d$.

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APPENDIX: EXTENSIONS OF THE BASELINE MODEL

This appendix outlines a few important extensions of the model analyzed in the main text. For a more detailed elaboration of these extensions, please refer to the discussion paper version of this article [Growiec (2007)].

A.1. HUMAN CAPITAL ACCUMULATION IN OLG MODELS

In overlapping-generations (OLG) models without intracohort heterogeneity, the age structure is simple and consists of two generations, young and old,[16] whose human capital stocks at t are h_t^Y and h_t^O, respectively. Now, our claim is that all OLG models in which human capital accumulation drives aggregate growth assume some form of human capital externalities. To show this, we divide the models presented in the literature into the following two groups.

The first group of models use a "trick" consisting effectively of assuming (implicitly or explicitly) that the child's human capital before schooling is equal to the parents' human capital after schooling, as if all parents' skills could be immediately transferred to their children upon birth [cf. Tamura (2001); de la Croix and Doepke (2003); and numerous other works],

$$h^O_{t+1} = h^Y_t f(\Xi_t), \qquad h^Y_t = h^O_t, \tag{A.1}$$

where the vector Ξ_t captures all factors in the human capital production function other than one's own human capital. However, such an assumption violates the intuitive requirement that human capital at birth be constant (or at least trendless) over time. This uneasy consequence is, however, frequently hidden by omitting the age superscript $i \in \{O, Y\}$, and by the subsequent lack of discussion of the level of human capital at birth.

The second group of models [e.g., Becker and Tomes (1986); Galor and Tsiddon (1997)] introduces human capital externalities via

$$h^O_{t+1} = \varphi\left(h^O_t, \Xi_t\right), \qquad h^Y_t \equiv h^Y, \tag{A.2}$$

where h^Y is constant over time. The human capital level of each consecutive generation when old is thus an (increasing) function of the human capital of its parents when old. In such models, the human capital of consecutive generations when old can grow without bound even though preschool human capital is fixed. This specification introduces intergenerational externalities explicitly.

Our crucial point here is that in the absence of intracohort heterogeneity, the inclusion of *parental* human capital in the human capital accumulation equation is dynamically equivalent to the inclusion of an externality from total (or average) human capital. Indeed, for population sizes of the young and the old denoted as N^O_t and N^Y_t, respectively, it is trivially obtained that parental human capital is an affine function of the average human capital in the society:

$$\bar{h}_t = \frac{N^Y_t h^Y + N^O_t h^O_t}{N^Y_t + N^O_t}. \tag{A.3}$$

In some papers [e.g., Rangazas (2000)], direct reference to the introduction of human capital externalities is made. In numerous others, though, the direct link between parental and average human capital is not mentioned.[17]

A useful exercise with OLG models is to look at the evolution of human capital levels under zero schooling effort. A simple microeconomic rationale suggests that it should be trendless.

A.2. REMOVING THE LINEARITY AT THE INDIVIDUAL LEVEL

The assumption that the increments to one's human capital are more than proportional to one's actual human capital level [cf. Belzil and Hansen (2002)] implies that there exists a finite age at which the individual's human capital reaches infinity (unless she quit school earlier). The converse assumption that these increments are less than proportional does not cause such explosivity problems (especially painful with the "perpetual youth" survival law, which does not impose an upper bound on people's ages).

The current generalization of equation (1) reads

$$\frac{d}{d\tau}h(t,\tau) = [\lambda l_h(\tau) + \mu l_Y(\tau)]\, h^{\phi}(t,\tau), \qquad \phi > 0, \quad \phi \neq 1, \tag{A.4}$$

with $h(t, 0) = h_0 > 0$. Solving this leads to a microlevel cross-sectional equation – human capital regressed on schooling effort and work experience – which takes a hyperbolic form instead of the log-linear (Mincerian) one:

$$h(t,\tau) = \left\{ h_0^{1-\phi} + (1-\phi)\left[\lambda \int_0^{\tau} l_h(s)ds + \mu \int_0^{\tau} l_Y(s)ds\right]\right\}^{\frac{1}{1-\phi}}. \tag{A.5}$$

The original Mincer equation is obtained only in the knife-edge case of $\phi = 1$.

Because $h(t, \tau)$ does not depend on t here, average human capital in a society whose age structure is stationary cannot change over time, just as is true in the baseline model.

A.3. MORE REALISTIC SURVIVAL LAWS

Let us now replace the unrealistic "perpetual youth" survival law with a more realistic one where an upper bound on people's lifespans exists.

As an illustrative example, we will use the realistic survival law put forward by Boucekkine et al. (2002) and further discussed by Azomahou et al. (in press). These authors assume that the unconditional probability of reaching an age of τ is given by

$$m(\tau) = \begin{cases} \dfrac{\alpha - e^{\beta\tau}}{\alpha - 1}, & \tau \leq \dfrac{\ln\alpha}{\beta} \\ 0, & \tau > \dfrac{\ln\alpha}{\beta}. \end{cases} \tag{A.6}$$

It is assumed that $\alpha > 1$ and $\beta > 0$. This survival law imposes an upper bound on people's lifespans: nobody can live longer than $M \equiv (\ln\alpha)/\beta$ years. The steady-state death rate d^* (cf. Appendix A.6) solves the implicit equation

$$d = \frac{\beta b}{\alpha - 1}\left(\frac{\alpha^{\frac{\beta-b+d}{\beta}} - 1}{\beta - b + d}\right). \tag{A.7}$$

Equation (A.7) offers two roots d^*, one of which is the spurious trivial root $d^* = b$, which must be neglected. The second, nontrivial root implies either a growing or a declining population: population will grow if

$$bE > 1 \quad \Leftrightarrow \quad \frac{b(\alpha\ln\alpha - \alpha + 1)}{\beta(\alpha - 1)} > 1 \tag{A.8}$$

and decline if $bE < 1$. If $bE = 1$, then the trivial root $d^* = b$ is unique, signifying a constant population.

We shall assume $l_h = \bar{l}_h \equiv$ const as well as $l_Y = \bar{l}_Y \equiv$ const for simplicity, and denote $\Phi = \lambda\bar{l}_h + \mu\bar{l}_Y$. Aggregating the individual human capital levels over the whole age structure of the society, we get

$$\begin{aligned}\bar{h}(t) &= \int_0^\infty be^{-(b-d^*)\tau} m(\tau) h(t-\tau,\tau)d\tau \\ &= \frac{bh_0}{\alpha-1}\left[\alpha\frac{e^{(\Phi-b+d^*)M}-1}{\Phi-b+d^*} - \frac{e^{(\Phi-b+d^*+\beta)M}-1}{\Phi-b+d^*+\beta}\right] \\ &= bh_0\left[\frac{\alpha\beta}{(\Phi-b+d^*+\beta)(\Phi-b+d^*)}\left(\frac{\alpha^{\frac{\Phi-b+d^*}{\beta}}-1}{\alpha-1}\right) - \frac{1}{\Phi-b+d^*+\beta}\right].\end{aligned} \tag{A.9}$$

Equation (A.9) is quite different from the hyperbolic equation (7), but it is not the Mincer equation either, so that the Mincerian log-linear earnings–schooling relationship is again lost upon aggregation.

Because $\bar{h}(t)$ is independent of t, human capital accumulation cannot be the engine of growth in the absence of knowledge spillovers, just as is true in the basic case.

A.4. TIME PROFILES OF EDUCATION AND WORK EFFORT

In this extension of the basic model, we will posit a more realistic (and more sophisticated) time profile of age-specific learning effort. We will now assume that at age τ, the individual assigns to formal schooling a share of time $l_h(\tau) = \theta/(\tau+\theta^{1/\psi})^\psi$, with $\theta > 0$ and $\psi \in (0,1)$. This implies that $l_h(0) = 1$ and from then on, learning effort declines according to a power law, gradually giving way to working: an individual aged τ devotes a share of time $l_Y(\tau) = 1 - \theta/(\tau+\theta^{1/\psi})^\psi$ to working.

Equation (2) is still obtained and it still means that log human capital depends linearly on total schooling effort and total work effort—the relevant integrals denote these cumulative measures. After integration, however, it is obtained that

$$w(t,\tau) = h(t,\tau) = h_0 \exp\left[\frac{(\lambda-\mu)\theta}{1-\psi}\left(\tau+\theta^{\frac{1}{\psi}}\right)^{1-\psi} + \tau - \frac{\lambda-\mu}{1-\psi}\theta^{\frac{1}{\psi}}\right]. \tag{A.10}$$

The expression in the exponent of equation (A.10) is linear in lifelong schooling effort, but it follows a *concave* shape in raw years of schooling, analogous to the one postulated by Bils and Klenow (2000). See the working paper version of this paper [Growiec (2007)] for a more precise elaboration of this claim.

Again, without human capital externalities, individuals' human capital $h(t,\tau)$ does not depend on t, and thus aggregate human capital $\bar{h}(t)$ is constant over time.

A.5. THE CASE OF TEACHERS HAVING PROPORTIONALLY MORE HUMAN CAPITAL

Let us now presume that formal education consists primarily in transferring knowledge from teachers to pupils and not in individual learning by the pupils (as in the pure externalities case $\theta = 0$). Suppose, however, that each pupil is taught by teachers who are older than she is by a constant number of years, say ω.[18] Equation (1) is then replaced by

$$\frac{d}{d\tau}h(t,\tau) = \lambda l_h(\tau)h(t,\tau+\omega) + \mu l_Y(\tau)h(t,\tau). \quad \textbf{(A.11)}$$

Under the assumptions that $l_h = \bar{l}_h \equiv$ const as well as $l_Y = \bar{l}_Y \equiv$ const, the characteristic equation of (A.11) has one or two positive real roots (growth rates of individual human capital) if ω is low enough, precisely if $\omega\mu\bar{l}_Y + \ln\omega + \ln(\lambda\bar{l}_h) + 1 \leq 0$. If they are violated, however, then equation (A.11) is not consistent with exponential growth and cannot be used to generate balanced growth of personal human capital as the individual ages.

Assuming balanced growth in individual human capital, we can exploit the exponentiality property and replace the assumption that the teacher is ω years older than the pupil with the assumption that the teacher has *proportionally more* human capital. Mathematically, this means replacing $h(t,\tau+\omega)$ with $mh(t,\tau)$, where $m > 1$.[19]

Using this "trick," equation (A.11) is quickly solved as

$$h(t,\tau) = h_0 \exp[(m\lambda\bar{l}_h + \mu\bar{l}_Y)\tau], \quad \textbf{(A.12)}$$

which implies only a "cosmetic" modification to (2), and the cross-sectional Mincerian relationship is preserved.

As $h(t,\tau)$ does not depend on t, we again conclude that the average level of human capital in the society will stay constant over time. Human capital externalities from $h(t,\tau+\omega)$ cannot be used to generate balanced growth in aggregate human capital. To achieve this, some form of externalities from aggregate human capital $\bar{h}(t)$ are necessary.

A.6. STATIONARY AGE STRUCTURE: DERIVATION

Whatever survival law $m(\tau)$ we choose, a stationary age structure implies that the death rate is constant and equal to d^*, where d^* is the nontrivial solution to the implicit equation

$$d + \int_0^\infty be^{(b-d)\tau}m'(\tau)d\tau = 0. \quad \textbf{(A.13)}$$

Equation (A.13) typically has two solutions. Of these two, only the nontrivial one is of economic interest.[20]

To characterize this solution more precisely, we denote the *life expectancy at birth* as

$$E = -\int_0^\infty \tau m'(\tau)d\tau. \quad \textbf{(A.14)}$$

Using this notation, we obtain the following result. If $bE > 1$, then the nontrivial solution of (A.13) implies that $b > d$, indicating a growing population. Conversely, if $bE < 1$, then $b < d$ holds, indicating a declining population. In the special case $bE = 1$, the zero growth solution $b = d$ is unique.[21]

This result is very intuitive: it means that population will grow steadily, preserving the shape of the age distribution, if and only if the average number of offspring per person is greater than one. Conversely, if the average number of offspring per person is less than one, the population will steadily decline. Obviously, the average number of offspring per person is directly bE here—the instantaneous fertility rate times the life expectancy.