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Factor-augmenting technology choice and monopolistic competition ☆

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ABSTRACT

We put forward a tractable, interpretable, and easily generalizable framework for modeling endogenous factor-augmenting technology choice by monopolistically competitive firms. The setup is framed within the standard Dixit and Stiglitz (1977) model of monopolistic competition. Optimal technology choice is made here either by final goods producers or the representative household which holds the shares of (differentiated) intermediate goods producers. These two cases have different implications for the distribution of output but they yield the same aggregate level of output, the same aggregate production function and equivalent macroeconomic dynamics. Thanks to this property, the proposed framework can be used as a building block in a variety of embedding structures, including those which require to be solved recursively (separately for the dynamics of aggregate variables and for the distribution in each time period).

1. Introduction

Back in the 1960s, the question of optimal choice of production techniques was at heart of the heated "Cambridge–Cambridge" controversy related to the underpinnings of the concept of physical capital (see e.g., Harcourt, 1972, for a summary). Later on, however, the macroeconomics profession has broadly agreed to base their further investigations either on the neoclassical assumption of a Cobb–Douglas aggregate production function or on the postulate of purely labor-augmenting technical change, given their analytical convenience, consistency with balanced growth (Uzawa, 1961) and a range of frequently invoked stylized facts (Kaldor, 1961). The question of endogenous technology choice has naturally lost its footing under such a paradigm. However, recent literature has uncovered ample evidence against the empirical validity of these assumptions (e.g., Klump et al., 2007; Hsieh and Klenow, 2009; Jones and Romer, 2010). Hand in hand with these findings came the renewed interest in endogenous technology choice (e.g., Jones, 2005; Caselli and Coleman, 2006; Nakamura and Nakamura, 2008; Nakamura, 2009): when the aggregate technology is not Cobb–Douglas, factor-augmenting technology choices become both theoretically important and empirically testable.

Given this background, the contribution of the current article is to propose a simple framework for modeling endogenous factor-augmenting technology choice in a monopolistic competition framework. In its core, it is a substantial

extension and reinterpretation of the model discussed recently by Growiec (2013), who used it to provide a microfoundation for the aggregate CES production function.[1] The key advantage of that approach lies with its ability to yield direct results on firms' optimal factor-augmenting technology choices, and that it naturally accomodates heterogeneity across sectors (with the symmetric case being an interesting benchmark) and factor-augmenting technical change (with Hicks-neutral technical change being a natural point of reference). It is also sufficiently tractable, interpretable, and generalizable to be available as a building block of a variety of embedding structures. In particular, it could help address questions regarding:

- Input misallocation and the related technical inefficiency across sectors and countries (Basu and Weil, 1998; Hsieh and Klenow, 2009).
- The determinants of the direction of technical change (Acemoglu, 2003; Caselli and Coleman, 2006; Klump et al., 2007; León-Ledesma et al., 2010).
- Firms' self-selection into international trade (Melitz, 2003; Bernard et al., 2003).
- Incentives in directed R&D and technology diffusion (Aghion and Howitt, 1998; Acemoglu, 2007), including the possibility of failures to switch to or coordinate on most efficient technologies.

In all these issues, endogenous technology choice is clearly an important part of the mechanism at work.

Following the direct predecessors of our current modeling approach, we allow firms to choose their preferred factor-augmenting technologies optimally from a parametrically specified technology menu whose shape is determined by the R&D sector (which we tentatively consider to be exogeneous here). This shape is isoelastic, consistent with the assumption that the underlying unit factor productivities (UFPs) are independently Weibull-distributed across the available technologies.[2] As opposed to the earlier contributions, however, the current article considers an economy with a *continuum* of monopolistically competitive producers of differentiated intermediate goods whose output is assembled into a unique consumption good using a CES technology, as in the standard model of monopolistic competition (Dixit and Stiglitz, 1977).[3] We consider two cases here: (1) with optimal technology choice on the side of perfectly competitive final goods producers (case FG) or (2) on the side of monopolistically competitive intermediate goods producers, represented by the representative household which holds a portfolio of their shares (case IG). The former one can be interpreted as one where the (labor-intensive) intermediate goods are essentially primitive, and all factor-augmenting technologies are embodied in the ingenuous ways these intermediates are assembled. Hence, case (FG) is a stylized representation of technological change in upstream industries (e.g., plastics, and semi-conductors), in R&D-intensive manufacturing sectors (e.g., ICT) or in services (e.g., financial intermediation). Case (IG) assumes, on the other hand, that assembling intermediates is a primitive process but the intermediates themselves are vehicles of (labor-saving) technological sophistication. Such a view of technological change is probably more reasonable for downstream industries (e.g., computer hardware, cars, and domestic appliances) or traditional manufacturing sectors.[4] We solve for the general equilibrium in both cases.

It turns out that both variants of the framework lead to *the same* aggregate production function – which takes the normalized CES form – and *identical* isoelastic technology choices. Prices and monopoly profits do not coincide, though: the identity of the vehicle of technological progress exerts a significant impact on the distribution of income between factors of production at any given point in time. On the other hand, the level and dynamics of aggregate output are unaffected. We view this "recursivity" property of our framework as very helpful in its prospective applications: one may first solve it separately for the dynamics of aggregate variables, and then separately for the distributions within each time period.

Viewed from a different perspective, the contribution of this paper to the literature can also be interpreted as a positive one. Indeed, in the current paper we have arguably identified two plausible mechanisms of endogenous technology choice at the firm level, which are able to disentangle the distribution of output across firms and production inputs from the aggregate quantities and their dynamics.

The remainder of the article is structured as follows. Section 2 lays out the framework. Section 3 presents the results. Section 4 concludes.

2. The framework

The description of the proposed framework is structured as follows. First we specify the technology menu from which factor-augmenting technologies are drawn. Next we present the optimization problems of both types of firms. Finally we discuss the possible closures of the model and define the equilibrium.

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[1] The *aggregate* (or "global") production function is obtained there as a convex hull of a range of "local" production functions. The considered problem refers to optimal technology choice by homogeneous firms, not to aggregation under heterogeneity. See also Jones (2005) and Growiec (2008a,b).

[2] See Growiec (2013) for a detailed justification of this distributional assumption as well as a fully specified model of the underlying R&D sector.

[3] Thanks to the tractability and transparency of its implications, the Dixit and Stiglitz (1977) model of monopolistic competition has become a cornerstone of contemporary economics. It is widely applied in theories of business cycles (e.g., Smets and Wouters, 2003), long-run growth (e.g., Romer, 1990), international trade (e.g., Melitz, 2003), industrial organization (e.g., d'Aspremont et al., 1996), as well as a wide range of other applications. For this reason we view it as a natural framework in which to consider endogenous factor-augmenting technology choice.

[4] One could also consider mixed cases where neither final nor intermediate goods producers have full control over the equilibrium technology choice. We leave this possibility for further research.

2.1. The technology menu

The technology menu is specified in the space of available ideas, i.e., collections of unit factor productivities (UFPs) denoted as a_i, where the subscript $i \in [0, A]$ indexes the intermediate goods sectors. The shape of this menu is determined by the R&D sector which we tentatively consider as exogenous:

Assumption 1. The technology menu, specified in the $\{a_i\}_{i \in [0,A]}$ space, is given by the equality:

$$H(a_i) = \int_0^A \left(\frac{a_i}{\lambda_{ai}} \right)^\alpha di = N, \qquad \lambda_{ai}, \alpha, N > 0, \tag{1}$$

where the factor-specific parameter λ_{ai} identifies the degree of augmentation of each ith intermediate good along the technology menu, α defines the curvature of the menu, and N defines its location.

The technology menu, parametrically specified above,[5] can be understood as a contour line of the cumulative distribution function of the joint distribution of unit factor productivities (UFPs) $\tilde{a}_i$. Under independence of all dimensions (so that marginal distributions are multiplied by one another), Eq. (1) obtains if and only if the marginal distributions are Weibull with the same shape parameter $\alpha > 0$ (Growiec, 2008b; Growiec, 2013):

$$P(\tilde{a}_i > a_i) = e^{-\left(\frac{a_i}{\lambda_{ai}} \right)^\alpha}, \qquad i \in [0, A], \tag{2}$$

where all $a_i > 0$. Under such parametrization, we have

$$P(\tilde{a}_i > a_i, \forall i \in [0, A]) = e^{-\int_0^A \left(\frac{a_i}{\lambda_{ai}} \right)^\alpha di}, \tag{3}$$

and thus the location parameter N in Eq. (1) is interpreted as $N = -\ln P(\tilde{a}_i > a_i, \forall i \in [0, A]) > 0$. Since each of the UFPs $\tilde{a}_i$ augments ith intermediate good directly, the technologies are also perfectly excludable and cannot be traded between sectors.

2.2. Final goods producers

Final goods producers assemble a continuum of measure A of differentiated, imperfectly substitutable intermediate goods x_i, indexed by $i \in [0, A]$, augmenting each of them with an optimally chosen factor-augmenting unit factor productivity (UFP) a_i. Intermediate goods are transformed into final output X according to the given "local" normalized[6] CES technology with constant returns to scale:

$$X = X_0 \left(\int_0^A \pi_{0i} \left(\frac{a_i x_i}{a_{0i} x_{0i}} \right)^\theta di \right)^{\frac{1}{\theta}}, \qquad \theta \in (0, 1), \tag{4}$$

where the variables with the subscript 0 are evaluated at the normalization point in time t_0, at which Eq. (4) is trivially satisfied. Variables without the subscript 0 are evaluated at any given point in time. $\theta \in (0, 1)$ is the *substitutability parameter*, related to the elasticity of substitution along the "local" technology via $\sigma_{LPF} = \frac{1}{1-\theta}$. In line with the literature on monopolistic competition, we assume that intermediate goods are gross substitutes.

The normalization constants π_{0i} are the shares of each ith intermediate good's remuneration in total output at time t_0. They must sum up to unity, consistently with constant returns to scale and their interpretation as shares:

$$\int_0^A \pi_{0i} di = 1. \tag{5}$$

Final goods producers are assumed to be perfectly competitive, and thus they decide upon their demand for intermediate goods x_i taking their prices q_i as given, leading to isoelastic demand curves for every ith intermediate good. In case (FG), they are also allowed to pick their favorite set of UFPs a_i, for all $i \in [0, A]$. In case (IG), this choice is left to intermediate goods producers, represented by their shareholders. One may say that in case (IG), all technological progress is thus embodied in intermediate goods, whereas in case (FG) it is embodied in the way they are assembled.

Assumption 2 (*Case FG*). Final goods firms choose the demanded quantities of intermediate goods and factor-augmenting technologies $(x_i, a_i)_{i \in [0, A]}$ optimally, subject to the current technology menu, taking prices $(q_i)_{i \in [0, A]}$ as given, such that their profit is maximized:

$$\max_{x_i, a_i \ \forall i \in [0, A]} \left\{ X_0 \left(\int_0^A \pi_{0i} \left(\frac{a_i x_i}{a_{0i} x_{0i}} \right)^\theta di \right)^{\frac{1}{\theta}} - \int_0^A q_i x_i di \right\} \quad s.t. \quad \int_0^A \left(\frac{a_i}{\lambda_{ai}} \right)^\alpha di = N. \tag{6}$$

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[5] For a formal definition and elaboration of the underlying R&D process, please refer to Growiec (2013).

[6] For a survey of the merits of normalization of CES production functions, see Klump et al. (2012).

Assumption 3 (*Case IG*). Final goods firms choose the demanded quantities of intermediate goods $(x_i)_{i\in[0,A]}$ optimally, taking prices and factor-augmenting technologies $(q_i, a_i)_{i\in[0,A]}$ as given, such that their profit is maximized:

$$\max_{x_i \ \forall i\in[0,A]} \left\{ X_0 \left(\int_0^A \pi_{0i} \left(\frac{a_i x_i}{a_{0i} x_{0i}} \right)^{\theta} di \right)^{\frac{1}{\theta}} - \int_0^A q_i x_i di \right\}. \tag{7}$$

Second order conditions require us to assume that $\alpha > \theta > 0$, so that the interior stationary point of the above optimization problem is a maximum. Moreover, for the resultant aggregate production function to be concave with respect to x_i, we need to assume also that $\alpha - \theta - \alpha\theta > 0$. In other words, the curvature parameter of the technology menu α must be large enough to exceed $\frac{\theta}{1-\theta}$.

Since final goods producers are perfectly competitive, in equilibrium their profits are zero, which implies:

$$X = \int_0^A q_i x_i di. \tag{8}$$

2.3. Intermediate goods producers

Intermediate goods producers are assumed to operate a linear technology using labor only, paying their employees the market wage w which they take as given. Each ith good is produced by a monopolist who can freely decide upon the price of her good q_i subject to the demand curve defined by final goods producers. In case (IG), this choice is then complemented with a choice of factor-augmenting technologies $(a_i)_{i\in[0,A]}$ such that the sum of profits earned by the intermediate goods sector is maximized.

Assumption 4 (*Case FG*). Intermediate goods firms indexed by $i \in [0,A]$ choose the intermediate goods price q_i optimally, subject to the demand curve $x_i(q_i)$ and taking $(a_i)_{i\in[0,A]}$ as given, such that their profit is maximized:

$$\max_{q_i}\{(q_i - w)x_i(q_i)\}, \tag{9}$$

considering the impact of their own choice q_i on aggregate output X as negligible.

Assumption 5 (*Case IG*). Intermediate goods firms indexed by $i \in [0,A]$ choose the intermediate goods price q_i optimally, subject to the demand curve $x_i(a_i, q_i)$ and taking $(a_i)_{i\in[0,A]}$ as given, such that their profit is maximized:

$$\max_{q_i}\{(q_i - w)x_i(a_i, q_i)\}, \tag{10}$$

considering the impact of their own choices (q_i) on aggregate output X as negligible.

The representative household – which holds the shares of all intermediate goods firms in its portfolio – maximizes the sum of their profits, taking the technology menu, the collection of demand curves $(x_i(a_i, q_i))_{i\in[0,A]}$ and the set of monopoly prices $(q_i)_{i\in[0,A]}$ as given:

$$\max_{a_i \ \forall i\in[0,A]} \left\{ \int_0^A (q_i - w)x_i(a_i, q_i) di \right\} \quad s.t. \quad \int_0^A \left(\frac{a_i}{\lambda_{ai}} \right)^{\alpha} di = N. \tag{11}$$

Monopolistically competitive intermediate goods producers achieve positive profits in equilibrium. Moreover, in case (IG), optimal technology choice made by the shareholders allows them to maximize the sum of these profits given the technology menu and the demand for intermediate goods coming from the final goods sector.[7]

2.4. Equilibrium

The model can be closed in numerous ways. Here we provide it with a simple static closure in order to display the salient features of the proposed framework as transparently as possible. Henceforth we assume that all output X is immediately consumed at all times t, so that $X_t \equiv C_t$. We also assume that technological progress takes the form of exogenous growth in the parameters λ_{ai}, governing the shape of the technology menu at each t.[8]

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[7] An equivalent specification could allow the intermediate goods firms to choose their preferred technology a_i by themselves, requiring them to pay royalties r_i to the patent owners (the representative household), proportional to a_i but not to x_i so that the marginal production cost remains unaffected. Patent owners would then adjust r_i across sectors accordingly, so that the total revenue from royalties is maximized. Other equivalent alternatives could also be proposed. In fact, technically speaking, what is needed to reproduce our results is only that the model features a mechanism which equalizes shadow prices of technologies a_i across all sectors $i \in [0,A]$.

[8] Alternatively one could, e.g., provide the model with a dynamic edge by assuming that households who own the monopolistically competitive intermediate goods producers, maximize the discounted stream of utility. They could also endogenously decide on the amount of R&D devoted to increasing each of λ_{ai}'s. There is also a possibility to introduce stochastic factors into the R&D sector, so that the distribution of λ_{ai}'s (and thus the distribution of all other variables) could be driven by a stochastic R&D process.

We also assume (tentatively) that labor supply is fixed and normalized to one, $L \equiv 1$, and adopt the following definition of equilibrium.

Definition 1. In case FG, the *equilibrium* is a collection $\{\{a_i\}_{i\in[0,A]}, \{x_i\}_{i\in[0,A]}, \{q_i\}_{i\in[0,A]}, w\}$, such that:

- Optimization problems (6) and (9) are solved.
- The wage rate w is set so that markets clear.

In case IG, the *equilibrium* is a collection $\{\{a_i\}_{i\in[0,A]}, \{x_i\}_{i\in[0,A]}, \{q_i\}_{i\in[0,A]}, w\}$, such that:

- Optimization problems (7), (10) and (11) are solved.
- The wage rate w is set so that markets clear.

3. Results

The results shall be presented in the following order. We begin with the derivation of the demand curve. The next step is to solve for the optimal technology choice and the aggregate production function. This is followed by the computation of supply and pricing of intermediate goods. We then solve for the general equilibrium from which we, finally, draw conclusions for the cross-sectional distributions and dynamics.

3.1. The demand curve

Solving for the first order conditions of the maximization problem of final goods firms, the following isoelastic demand curves are derived:

Proposition 1. (Case FG). *If final goods producers pick $(a_i)_{i\in[0,A]}$ optimally, then their demand curve for intermediate goods takes the form:*

$$x_i(q_i) = \left[\frac{1}{q_i} X_0^{\frac{\alpha\theta}{\alpha-\theta}} X^{\frac{\alpha-\theta-\alpha\theta}{\alpha-\theta}} \pi_{0i} \left(\frac{\lambda_{ai}}{x_{0i}\lambda_{a0i}} \right)^{\frac{\alpha\theta}{\alpha-\theta}} \right]^{\frac{\alpha-\theta}{\alpha-\theta-\alpha\theta}}. \tag{12}$$

Proposition 2. (Case IG). *If the intermediate goods sector picks $(a_i)_{i\in[0,A]}$ optimally, then final good producers' demand curve for intermediate goods takes the following form, dependent on a_i:*

$$x_i(a_i, q_i) = \left[\frac{1}{q_i} X_0^\theta X^{1-\theta} \pi_{0i} \left(\frac{a_i}{x_{0i}a_{0i}} \right)^\theta \right]^{\frac{1}{1-\theta}}. \tag{13}$$

3.2. Optimal technology choice

The proposed framework provides direct results on the optimal factor-augmenting technology choices. Curiously, owing to the isoelastic character of the demand curves derived above, the independence of the technology menu of x_i and q_i, and the assumed lack of impact of technology choices on marginal production costs of intermediate goods producers, these choices are *exactly the same* regardless of who makes them, i.e., whether the technologies are embodied in intermediate goods or in the methods of assembling them.

More precisely,[9] for both cases (FG) and (IG) we find that at time t_0, when $X = X_0$, $x_i = x_{0i}$, and $\lambda_{ai} = \lambda_{a0i}$ for all $i \in [0,A]$, the optimal technology choice satisfies:

$$a_{0i}^* = (N\pi_{0i})^{\frac{1}{\alpha}} \lambda_{a0i}, \qquad i \in [0,A], \tag{14}$$

where λ_{a0i} is the value of λ_{ai} at time t_0. Values of a_{0i}^* will be used as a_{0i} in the normalization at the local level in all subsequent derivations. Keeping this normalization assumption in mind, we find the following result:

Proposition 3. (Cases FG and IG). *For any moment in time $t \neq t_0$, the optimal technology choices are:*

$$\left(\frac{a_j}{a_{0j}} \right)^* = \frac{\lambda_{aj}}{\lambda_{a0j}} \left(\int_0^A \pi_{0i} \left(\frac{\lambda_{ai}}{\lambda_{aj}} \frac{\lambda_{a0j}}{\lambda_{a0i}} \frac{x_i x_{0j}}{x_{0i} x_j} \right)^{\frac{\alpha\theta}{\alpha-\theta}} di \right)^{-\frac{1}{\alpha}}, \tag{15}$$

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[9] Derivation of Eqs. (14) and (15) in case (IG) requires one to solve for the supply and pricing of intermediate goods first.

for all $j \in [0,A]$.

At this point, we shall make the following definition which will be useful in our further calculations:

$$\Phi_j \equiv \int_0^A \pi_{0i} \left(\frac{\lambda_{ai}}{\lambda_{aj}} \frac{\lambda_{a0j}}{\lambda_{a0i}} \frac{x_i x_{0j}}{x_{0i} x_j} \right)^{\frac{\alpha\theta}{\alpha-\theta}} di. \tag{16}$$

The term Φ_j can be interpreted as the inverse of the *relative* supply of *j*th intermediate good as compared to all other goods, specified in *efficient* units, i.e., in units proportional to the parameter λ_{aj} describing the augmentation of *j*th good along the technology menu. In the symmetric case where $\Phi_i = \Phi_j$ for any $i \neq j$, Φ_j reduces to unity. This term will be used later to determine the lack of impact of dispersion across sectors on the aggregate variables and their dynamics: Φ_j defines the distributions of all quantities across firms but disappears upon aggregation.

3.3. The aggregate production function

Inserting the optimal technology choices, derived in the previous subsection, into the "local" production technology, i.e., computing the convex hull of "local" production functions given by Eq. (4), we obtain the following aggregation result.[10]

Proposition 4. (Cases FG and IG). *The aggregate production function, taking optimal factor-augmenting technology choices into account, is of normalized CES form:*

$$X = X_0 \left(\int_0^A \pi_{0i} \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \frac{x_i}{x_{0i}} \right)^{\frac{\alpha\theta}{\alpha-\theta}} di \right)^{\frac{\alpha-\theta}{\alpha\theta}}. \tag{17}$$

This aggregation result has three crucial properties. First, the aggregate production function inherits the CES form of the "local" production technology (Growiec, 2013). Second, it implies that the aggregate elasticity of substitution $\sigma = \frac{\alpha-\theta}{\alpha-\theta-\alpha\theta}$ unambiguously exceeds the "local" one, $\sigma_{LPF} = \frac{1}{1-\theta}$. It follows that intermediate goods are gross substitutes not only along the "local" production function, but also along the aggregate production function. Moreover, endogenous technology choice adds a further margin of substitution as compared to the "local" technology, and hence intermediate goods are even more easily substitutable in the aggregate than locally. Third, normalization with parameters π_{0i} can be maintained simultaneously at the local and the aggregate level. We also note the following corollary.

Corollary 1. *The income share of each ith intermediate good is equal to:*

$$\pi_i = \frac{\pi_{0i} \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \frac{x_i}{x_{0i}} \right)^{\frac{\alpha\theta}{\alpha-\theta}}}{\int_0^A \pi_{0i} \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \frac{x_i}{x_{0i}} \right)^{\frac{\alpha\theta}{\alpha-\theta}} di}, \qquad i \in [0,A]. \tag{18}$$

It is a substantial analytical advantage of the current framework that it disentangles endogenous factor-augmenting technology choice from the choice of quantities x_i and prices q_i. In addition, the former is also independent of who picks the UFPs a_i, i.e., whether the monopoly power enjoyed by intermediate goods producers extends to the freedom of technology choice or not. This result is a consequence of the isoelastic character of the derived demand curves, the independence of the technology menu of x_i and q_i, and the assumed lack of impact of technology choices on marginal production costs of intermediate goods producers.

3.4. Supply and pricing of intermediate goods

Solving for the first order condition of the profit maximization problem of each *i*th intermediate goods producer, subject to the demand curve, we obtain the following optimal prices and supplied quantities:

Proposition 5. (Case FG). *If final goods producers pick $(a_i)_{i \in [0,A]}$ optimally, then*

$$q_i = \left(\frac{\alpha-\theta}{\alpha\theta} \right) w, \qquad \frac{x_i}{x_{0i}} = \frac{X}{X_0} \frac{1}{\Phi_i} \frac{w_0}{w}. \tag{19}$$

Proposition 6. (Case IG). *If the intermediate goods sector picks $(a_i)_{i \in [0,A]}$ optimally, then*

$$q_i = \frac{w}{\theta}, \qquad \frac{x_i}{x_{0i}} = \frac{X}{X_0} \frac{1}{\Phi_i} \frac{w_0}{w}. \tag{20}$$

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[10] Please consult the Appendix in Growiec (2013) for proof.

One natural result (cf. Dixit and Stiglitz, 1977) is that since all monopolists pay their workers the same equilibrium wage w, they will also necessarily demand the same price q_i for a unit of the produced intermediate good, $q_i = q_j$ for all $i \neq j$. It is also not surprising that this price is set at a fixed proportional markup over the monopolist's marginal cost w.

Consequently, optimal monopoly profits satisfy $z_i = \frac{\alpha - \theta - \alpha\theta}{\alpha\theta} w x_i$ in case (FG) and $z_i = \frac{1-\theta}{\theta} w x_i$ in case (IG). Thus in both cases we obtain $\frac{z_i}{z_{0i}} = \frac{X}{X_0} \frac{1}{\Phi_i}$. In consequence, at each moment in time monopoly profits are proportionally larger in case (IG) than in case (FG). Across time, however, the evolution of profits (and output) is exactly parallel, so that the ratios $\frac{z_i}{z_{0i}}$ and $\frac{x_i}{x_{0i}}$ are equal in both cases.

Finally, looking at cross-demand $\frac{x_i}{x_j}$ enables us to establish that $\frac{x_{0i}}{x_{0j}} = \frac{\pi_{0i}}{\pi_{0j}} = \frac{z_{0i}}{z_{0j}}$ which, coupled with the assumption of a fixed labor supply, $\int_0^A x_i di = L \equiv 1$, leads to the conclusion that $x_{0i} = \pi_{0i}$ for all $i \in [0, A]$.

3.5. Equilibrium

Let us now compute the equilibrium wage w and infer all remaining aggregate quantities. From the equilibrium condition $X = \int_0^A q_i x_i di$, holding for all t, we obtain the following:

Proposition 7. *The wage rate equals:*

$$w = \left(\frac{\alpha\theta}{\alpha - \theta} \right) X, \tag{21}$$

in case (FG) and

$$w = \theta X, \tag{22}$$

in case (IG).

Thus, taking the normalization moment in time t_0, it is also true that $w_0 = \left(\frac{\alpha\theta}{\alpha - \theta}\right) X_0$ in case (FG) and $w_0 = \theta X_0$ in case (IG). Consequently, for both cases we have the following dynamic relationship:

$$\frac{w}{w_0} = \frac{X}{X_0}. \tag{23}$$

It follows that the wage rate is proportionally larger in case (FG) than in case (IG). This is not surprising since in case (IG), monopoly power of intermediate goods producers – who are also the only employers of labor in this model – is extended also to technology choice, which is then used as an additional tool for extracting the surplus from the employees. The dynamic evolution of wages is exactly parallel in both cases, though.

As far as monopoly profits are concerned, we obtain:

Proposition 8. *The aggregate monopoly profit of intermediate goods producers satisfies:*

$$Z = \int_0^A z_i di = \left(\frac{\alpha - \theta - \alpha\theta}{\alpha - \theta} \right) X, \tag{24}$$

in case (FG) and

$$Z = \int_0^A z_i di = (1 - \theta) X, \tag{25}$$

in case (IG).

Hence in both cases markets clear, so that the entire output X is distributed between the remuneration of workers and monopoly profits, only that the share of the latter is relatively larger in case (IG):

$$X = Z + wL = Z + w. \tag{26}$$

Also, the dynamic evolution of aggregate monopoly profit exactly parallels the evolution of aggregate output:

$$\frac{Z}{Z_0} = \frac{X}{X_0}. \tag{27}$$

Looking at cross-demand $\frac{x_i}{x_j}$ again and taking the above aggregate results into account, it is easily verified that:

$$\frac{x_i}{x_j} = \frac{\pi_i}{\pi_j} = \frac{z_i}{z_j} = \frac{\pi_{0i}}{\pi_{0j}} \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \frac{\lambda_{a0j}}{\lambda_{aj}} \right)^{\frac{\alpha\theta}{\alpha - \theta - \alpha\theta}}. \tag{28}$$

Using these optimal supply decisions and the inverse relative efficient supply of respective intermediate goods Φ_i, we find that the normalized aggregate variables take the same values irrespective of the vehicle of technological progress, i.e., in both cases (FG) and (IG):

Proposition 9. (Cases FG and IG). *In equilibrium, aggregate output can be rewritten in terms of technology coefficients λ_{ai} and normalization constants only:*

$$X = X_0 \left(\int_0^A \pi_{0i} \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \right)^{\frac{\alpha\theta}{\alpha-\theta-\alpha\theta}} di \right)^{\frac{\alpha-\theta-\alpha\theta}{\alpha\theta}} = X_0 \left(\int_0^A \pi_{0i} \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \right)^{\sigma-1} di \right)^{\frac{1}{\sigma-1}}. \tag{29}$$

The shares of intermediate goods π_i satisfy:

$$\pi_i = \frac{\pi_{0i} \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \right)^{\frac{\alpha\theta}{\alpha-\theta-\alpha\theta}}}{\int_0^A \pi_{0i} \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \right)^{\frac{\alpha\theta}{\alpha-\theta-\alpha\theta}} di} = \frac{\pi_{0i} \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \right)^{\sigma-1}}{\int_0^A \pi_{0i} \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \right)^{\sigma-1} di}, \qquad i \in [0,A]. \tag{30}$$

The supplied quantities of intermediate goods satisfy:

$$\frac{x_i}{x_{0i}} = \frac{\pi_i}{\pi_{0i}} = \frac{1}{\Phi_i} = \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \right)^{\frac{\alpha\theta}{\alpha-\theta-\alpha\theta}} \left(\frac{X}{X_0} \right)^{-\frac{\alpha\theta}{\alpha-\theta-\alpha\theta}} = \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \right)^{\sigma-1} \left(\frac{X}{X_0} \right)^{1-\sigma}. \tag{31}$$

Monopoly profits are equal to:

$$\frac{z_i}{z_{0i}} = \frac{x_i}{x_{0i}} \frac{X}{X_0} = \frac{1}{\Phi_i} \frac{X}{X_0} = \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \right)^{\frac{\alpha\theta}{\alpha-\theta-\alpha\theta}} \left(\frac{X}{X_0} \right)^{\frac{\alpha-\theta-2\alpha\theta}{\alpha-\theta-\alpha\theta}} = \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \right)^{\sigma-1} \left(\frac{X}{X_0} \right)^{2-\sigma}. \tag{32}$$

This summarizes the equilibrium of the current model. Let us now draw two important conclusions from it, pertaining to cross-sectional distributions and economic dynamics. Both of them are interpretable in terms of the recursivity property of the model.

3.6. Cross-sectional distributions

As is apparent from Eq. (28), the framework provides clear predictions on cross-sectional distributions of all variables. More precisely, we find that the distributions of x_i, z_i, π_i, a_i are determined in a crucial way by two elements:

- The initial distribution $(\pi_{0i})_{i\in[0,A]}$.
- The relative pace of factor-augmenting technological progress, expanding the available technology menu, captured by growth of λ_{ai} relative to λ_{aj}, where $j \neq i$.

Quite naturally, if the initial distribution is uniform, $\pi_{0i} = \pi_{0j}$ for all $i \neq j$, then the initial distribution of all key variables of the model is uniform as well, and all heterogeneity across sectors must come from the second channel. Conversely, if the growth rate of all λ_{ai}'s is the same, and thus accounting for endogenous technology choice we have Hicks-neutral technical change in equilibrium, then the initial distribution $\frac{x_{0i}}{x_{0j}} = \frac{\pi_{0i}}{\pi_{0j}} = \frac{z_{0i}}{z_{0j}}$ is maintained for all t, so that all heterogeneity must come from the first channel. Otherwise, if technical change is not Hicks-neutral, then the cross-sectional distributions will evolve over time, driven uniquely by the evolution of $(\Phi_i)_{i\in[0,A]}$.

3.7. Dynamics

The framework also provides clear predictions on the growth rates of aggregate variables as well as their disaggregate counterparts, based on Eqs. (23), (27), (29), (31), and (32). We find that the growth rates of $X, Z, w, x_i, z_i, \pi_i, a_i$ are determined in a crucial way by two elements:

- The *average* rate of factor-augmenting technological progress (as captured by $\tilde{\lambda}_a$, the average growth rate of λ_{ai}'s) which determines the growth rate of all aggregate variables: output X, wage w, and total monopoly profit Z.
- The *relative* pace of technological progress augmenting each specific ith good (as captured by the growth rate of λ_{ai} relative to $\tilde{\lambda}_a$), which determines the relative rise or fall in x_i, z_i, π_i, a_i compared to the economy-wide average.

Indeed, according to Eq. (29), the growth rate of output X (and thus aggregate profits Z and wages w) is proportional to the growth rate of the technological augmentation parameters λ_{ai}, averaged over the interval $i \in [0,A]$. To see this, note that from the Cauchy intermediate value theorem it follows that there exists an average growth rate $\frac{\tilde{\lambda}_a}{\tilde{\lambda}_{a0}}$, for which the following is satisfied:

$$\frac{X}{X_0} = \frac{Z}{Z_0} = \frac{w}{w_0} = \left(\int_0^A \pi_{0i} \left(\frac{\lambda_{ai}}{\lambda_{a0i}} \right)^{\sigma-1} di \right)^{\frac{1}{\sigma-1}} = \left(\int_0^A \pi_{0i} \left(\frac{\tilde{\lambda}_a}{\tilde{\lambda}_{a0}} \right)^{\sigma-1} di \right)^{\frac{1}{\sigma-1}} = \frac{\tilde{\lambda}_a}{\tilde{\lambda}_{a0}}. \tag{33}$$

Hence, the average rate of factor-augmenting technical change $\frac{\dot{\tilde{\lambda}}_a}{\tilde{\lambda}_{a0}}$ alone pins down the growth rate of all aggregate quantities in our framework. Let us now turn to the disaggregate variables. First, if all parameters λ_{ai} grow at the same rate, and thus we have Hicks-neutral technical change in equilibrium, then for all $i \in [0,A]$ we have:

$$\frac{\lambda_{ai}}{\lambda_{a0i}} = \frac{\tilde{\lambda}_a}{\tilde{\lambda}_{a0}} \quad \Rightarrow \quad \left(\frac{x_i}{x_{0i}} = \frac{\pi_i}{\pi_{0i}} = \frac{1}{\Phi_i} = 1 \quad \wedge \quad \frac{z_i}{z_{0i}} = \frac{a_i}{a_{0i}} = \frac{X}{X_0} = \frac{\tilde{\lambda}_a}{\tilde{\lambda}_{a0}} \right), \tag{34}$$

so that the second channel is switched off. Please note that the supply of each intermediate good, expressed in physical terms (x_i) is constant in such case, due to the assumption of a constant labor input. The supplied *value* of intermediate goods ($q_i x_i$), their prices q_i and UFPs a_i increase over time, however, provided that the average rate of factor-augmenting technological progress $\frac{\dot{\tilde{\lambda}}_a}{\tilde{\lambda}_{a0}}$ is positive.

Otherwise, if technical change is not purely Hicks-neutral in equilibrium, then the dynamics of the quantity of each intermediate good produced x_i as well as the respective monopoly profits z_i will follow directly the dynamics of its factor share π_i. All of them will simultaneously increase if and only if λ_{ai} grows faster than average, and decrease if λ_{ai} grows slower than average.

4. Conclusion

We have proposed a simple framework for modeling endogeneous factor-augmenting technology choice in an economy with monopolistic competition à la Dixit and Stiglitz (1977). The framework provides direct results on firms' optimal factor-augmenting technology choices. It is also tractable, interpretable, and easily generalizable. One of its key strengths lies with its "recursivity" property: one may solve it separately for the optimal technology choices and the dynamics of aggregate variables, and then separately for cross-sectional distributions within each time period. Furthermore, the identity of the vehicle of technical change, or equivalently of the entity who makes the optimal technology choice, exerts a significant impact on the distribution of income at a given point in time but not on the aggregate output level, the aggregate production function, nor on the macroeconomic dynamics.

For these reasons, we believe it could be easily used as a building block in a range of models aimed at addressing diverse economic questions, related, among others, to the issues of input misallocation, determinants of the direction of technical change, firms' self-selection into international trade, and the incentives in directed R&D and technology diffusion.

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