	Testing for integration	Cointegration	
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Lecture 9: Nonstationarity. Error Correction Models

Econometric Methods

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Outline



- Stationarity and nonstationarity
 - Notion of stationarity
 - Random walk as nonstationary time series
- 2 Testing for integration
 - Dickey-Fuller test
 - Augmented D-F specification
- Cointegration
- Error correction model

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2 Testing for integration

3 Cointegration

4 Error correction model

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Stationarity and nonstationarity	Testing for integration	Cointegration	
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Notion of stationarity			

Time series – definition

 Y_t - random variable - takes values with some probabilities (described by density function f(y)/ distribution function F(y)) values + probabilities: distribution of a random variable { Y_t } - stochastic process - sequence of random variables Y_t ordered by time

 $\{y_t\}$ – time series – draw from a stochastic process in one sample

Key parameters of random variable's distribution:

Expected value: $E(Y) = \int_{-\infty}^{+\infty} yf(y) dy$ Variance: $D^2(Y) = E(Y - E(Y))^2$

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Notion of stationarity			

Stationarity

Type I (in the strict sense / strong)

Distribution of the stochastic process is time-invariant (in every period, y_t is a value taken by identically distributed random variable Y_t)

Type II (in the large sense / weak)

- mean and variance constant over time $E(Y_t) = \mu < \infty D^2(Y_t) = \sigma^2 < \infty$ - covariance between variables depends on their distance in time (not the moment in time) $Cov(Y_t, Y_{t+h}) = Cov(Y_{t+k}, Y_{t+k+h}) = \gamma(h)$

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Stationarity and nonstationarity	Testing for integration	Cointegration	
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Notion of stationarity			

"White noise"

"White noise" – definition

 $\begin{array}{ll} E(\varepsilon_t)=0 & [\mbox{fluctuations around zero}] \\ D^2(\varepsilon_t)=\sigma^2<\infty & [\mbox{homoskedasticity}] \\ Cov(\varepsilon_t,\varepsilon_{t+h})=0, \ h\neq 0 & [\mbox{no serial correlation}] \\ ... \mbox{like the disturbances in the classical linear regression model.} \end{array}$

- $\varepsilon_t \sim IID(0, \sigma^2)$
- I independent
- I indentically
- D distributed

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Notion of stationarity

Stationary series - "white noise"



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Nonstationary process - "random walk"

Random walk - definition

$$y_{t} = y_{t-1} + \varepsilon_{t}$$

$$y_{t} = y_{t-1} + \varepsilon_{t} = y_{t-2} + \varepsilon_{t-1} + \varepsilon_{t} = y_{t-3} + \varepsilon_{t-2} + \varepsilon_{t-1} + \varepsilon_{t} = \dots =$$

$$y_{0} + \sum_{t=1}^{T} \varepsilon_{t} \xrightarrow{y_{0}=0} \sum_{\substack{t=1\\t=1}}^{T} \varepsilon_{t}$$

stochastic trend

Properties of random walk:

$$E(y_t) = E(\sum_{t=1}^{T} \varepsilon_t) = \sum_{t=1}^{T} E(\varepsilon_t) = 0$$

$$D^2(y_t) = D^2(\sum_{t=1}^{T} \varepsilon_t) \stackrel{cov(\varepsilon_t, \varepsilon_{t-h})=0}{=} \sum_{t=1}^{T} D^2(\varepsilon_t) = T\sigma^2$$

$$cov(y_t, y_{t-h}) = E(y_t, y_{t-h}) - E(y_t)E(y_{t-h}) =$$

$$E(\sum_{t=1}^{T-h} \varepsilon_t \sum_{t=1}^{T-h} \varepsilon_t) - E(\sum_{t=1}^{T-h})E(\sum_{t=1}^{T-h}) = D^2(\sum_{t=1}^{T-h} \varepsilon_t) = \sum_{t=1}^{T-h} D^2(\varepsilon_t) = (T-h)\sigma^2$$

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Nonstationary series - random walk



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Random walk as nonstationary time series			

Integration order

Stationary variable is called integrated of order 0, notation: I(0).

Integration order – definition

Variable y_t is integrated of order d ($y_t \sim I(d)$) if it can be transformed into a stationary variable after differencing d times.

E.g. variable y_t generated by the process $y_t = y_{t-1} + \varepsilon_t$ is integrated of order 1 $(y_t \sim I(1))$, as $y_t - y_{t-1} = \varepsilon_t$, and $\varepsilon_t \sim I(0)$ by definition.

First differences: $\Delta y_t = y_t - y_{t-1}$

Second differences:

 $\Delta \Delta y_t = \Delta y_t - \Delta y_{t-1} = y_t - y_{t-1} - y_{t-1} + y_{t-2} = y_t - 2y_{t-1} + y_{t-2}$ ("filter (1,-2,1)")

Stationarity and nonstationarity		Cointegration	
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Nonstationary process - random walk with drift

Random walk with drift - definition

$$y_{t} = \alpha_{0} + y_{t-1} + \varepsilon_{t}$$

$$y_{t} = \alpha_{0} + y_{t-1} + \varepsilon_{t} = \alpha_{0} + \alpha_{0} + y_{t-2} + \varepsilon_{t-1} + \varepsilon_{t} =$$

$$\alpha_{0} + \alpha_{0} + \alpha_{0} + y_{t-3} + \varepsilon_{t-2} + \varepsilon_{t-1} + \varepsilon_{t} = \dots \stackrel{y_{0}=0}{=} T\alpha_{0} + \sum_{t=1}^{T} \varepsilon_{t}$$

stochastic trend

Random walk with drift - properties:

 $E(y_t) = E(T\alpha_0 + \sum_{t=1}^T \varepsilon_t) = T\alpha_0 + \sum_{t=1}^T E(\varepsilon_t) = T\alpha_0$

Variance and covariance – the same as in the case of random walk (adding a constant does not affect the dispersion).

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Stationarity and nonstationarity	Testing for integration	Cointegration	
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Nonstationary variable - random walk with drift



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Stationarity and nonstationarity	esting for integration	Cointegration	Error correction model
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Order of integration – why it matters

• possible problems:

- regression I(1) vs I(1) **spurious regression** (trending variables)
- regression I(0) vs I(1) nonstationary residuals (economically unlikely), mistakes in statistical inference (test statistic for variable significance is not t-distributed)

solutions:

- transformation of the series (differencing, logarithm)
- respecification of the model (for consistency with the theory)
- error correction and cointegration

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Dickey-Fuller test			

Dickey-Fuller (DF)

Dickey and Fuller, 1979, 1981

 $y_t = \alpha_1 y_{t-1} + \varepsilon_t$

- process is stationary if $\alpha_1 < 1$
- process is nonstationary if $\alpha_1 = 1$ (then random walk)

 $H_0: \alpha_1 = 1$

 $H_1: \alpha_1 < 1$

 true H₀ → nonstationary variables → bias in OLS → hypothesis not verifiable immediately

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Dickey-Fuller test			

DF – test statistic

Subtract y_{t-1} from both sides – potentially stationary dependent variable:

$$\Delta y_t = \underbrace{(\alpha_1 - 1)}_{\delta} y_{t-1} + \varepsilon_t$$

$$H_0: \ \delta = 0 \iff \alpha_1 = 1 \iff y_t \sim I(1)$$

$$H_1: \ \delta < 0 \iff \alpha_1 < 1 \iff y_t \sim I(0)$$

$$DF^{emp} = \frac{\hat{\delta}}{\hat{S}_{\delta}} \sim DF$$

(computed as t-statistic, but with different distribution – see *MacKinnon (1996)*)

If $DF^{emp} < DF^*$, reject H_0 against H_1 and consider the process stationary.

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Dickey-Fuller test			

DF – algorithm

 H_0 not rejected:

• nonstationarity, but unknown order of integration (1 or higher)...

Test again...

 variable is integrated of order 2 (y_t ~ I(2)), if stationary after double differentiation

$$\begin{split} \Delta(\Delta y_t) &= \delta \Delta y_{t-1} + \varepsilon_t \\ H_0: \ \delta &= 0 \Longleftrightarrow y_t \sim I(2) \\ H_1: \ \delta < 0 \Longleftrightarrow y_t \sim I(1) \\ DF^{emp} &= \frac{\hat{\delta}}{\hat{S}_{\delta}} \sim DF \end{split}$$

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Dickey-Fuller test			

DF - I(2) series

If again H_0 not rejected:

• variable integrated of order 2 or not rejecting H_0 due to low power of the test...

Test again...

$$\Delta^{3} y_{t} = \delta \Delta^{2} y_{t-1} + \varepsilon_{t}$$

$$H_{0} : \delta = 0 \iff y_{t} \sim I(3)$$

$$H_{1} : \delta < 0 \iff y_{t} \sim I(2)$$

$$DF^{emp} = \frac{\hat{\delta}}{\hat{S}_{\delta}} \sim DF$$

- failure to reject the null again too weak power of the test (too rarely rejects false null hypothesis)
- in economics, series integrated of order higher than 2 generally absent

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Augmented D-F specification			
Augmented DF			

Said and Dickey, 1985

- The power of DF test substantially lower under serial correlation of residuals in the test regression.
- This serial correlation should be handled.
- The simplest solution: dynamise the model by supplementing lags of the dependent variable.

Augmented Dickey-Fuller test - ADF

 $\Delta y_t = \delta y_{t-1} + \gamma_1 \Delta y_{t-1} + \gamma_2 \Delta y_{t-2} + \ldots + \gamma_k \Delta y_{t-k} + \varepsilon_t$

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Augmented D-F specification			

ADF – how many lags?

- in general: the purpose is to eliminate the serial correlation of the error term
- CAUTION! we do not use DW statistic to evaluate it (remember why...?)
- auxiliary algorithms: set the maximum lag length to consider and...
 - ...pick the best regression by means of information criteria (AIC, SIC, HQC)
 - $\bullet\,$...see if the last significant; if not, remove it and check again
- rule-of-thumb formula for maximum lag length: $(4 \cdot \frac{T}{100})^{\frac{1}{4}}$, where T sample size (*Schwert, 2002*)

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Augmented D-F specification

ADF - specification of test regression

$$\Delta y_t = \delta y_{t-1} \left(+ \sum_{i=1}^k \gamma_i \Delta y_{t-i} \right) + \varepsilon_t$$

possible extensions:

$$\Delta y_t = \frac{\beta}{\beta} + \delta y_{t-1} + \left(\sum_{i=1}^k \gamma_i \Delta y_{t-i}\right) + \varepsilon_t$$

 \rightarrow $\textit{H}_{0}:$ nonstationary process with drift

additionally: linear trend, quadratic trend, seasonal dummies...

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Augmented D-F specification

(A)DF – trendstationarity

$$\Delta y_t = \frac{\beta}{\beta} + \delta y_{t-1} + \left(\sum_{i=1}^k \gamma_i \Delta y_{t-i}\right) + \varepsilon_t$$

extension:

$$\Delta y_t = \beta + \delta y_{t-1} + \left(\sum_{i=1}^k \gamma_i \Delta y_{t-i}\right) + \frac{\gamma t}{\varepsilon_t} + \varepsilon_t$$

 \rightarrow if H_0 rejected and trend *t* significant – trendstationary series (then include *t* in the regression)

 \rightarrow if H_0 not rejected, series is nonstationary

Beware the difference!

stationary process \neq deterministic trend \neq stochastic trend

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Augmented D-F specification			
Exercise (1)			

Test the order of integration of the following variables with ADF test:

- log real wages
- log labour productivity
- unemployment

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Outline



2 Testing for integration

3 Cointegration



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24 / 32

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Nonstationarity - what then?

• dealing with models based on nonstationary variables:

- OLS: spurious regression
- explosive behaviour of dynamic models
- I(0)+I(1): nonstationary residuals + false statistical inference

solution: differencing l(1) data

- OLS, dynamic models: adequate tools
- LONG-TERM RELATIONSHIPS lost from the data

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Nonstationarity - what then?

• dealing with models based on nonstationary variables:

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Cointegration of time series

- dependencies between nonstationary variables sometimes stable in time
- ...then called COINTEGRATING RELATIONSHIPS
- there is a mechanism that brings the system back to equilibrium everytime it is shocked away from it (Granger theorem)

Cointegration

Time series y_1 and y_2 are cointegrated (of order d, b), if they are integrated of order d and there exists their linear combination integrated of order d - b: $y_1, y_2 \sim CI(d, b) \iff y_1 \sim I(d) \land y_2 \sim I(d) \land \exists_{\beta \neq 0} \quad \underbrace{y^T \beta}_{y_1 \beta_1 + y_2 \beta_2} \sim I(d - b)$ Usually: variables integrated of order 1, their combination –

stationary.

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Testing for cointegration: Engle-Granger procedure

Test the order of integration.

- If all of them, say, I(1), specify the cointegrating relationship: y_{1t} = β₀ + β₁y_{2t} + ε_t
 - **1** estimate parameters β_0, β_1 via OLS
 - **2** compute the residual series $(\hat{\epsilon}_t)$
- Test these residuals for stationarity if stationary, variables are cointegrated.

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- Test the order of integration.
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Outline



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3 Cointegration



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Error correction model (1)

• some "error correction" mechanism directly implied by the Granger theorem

• recall the ADL(1,1) model:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \beta_0 x_t + \beta_1 x_{t-1} + \varepsilon_t$$

 $y_t = \alpha_0 + \alpha_1 y_{t-1} + \beta_0 x_t + \beta_1 x_{t-1} + \varepsilon_t$
 $\varepsilon_t + y_{t-1} - y_{t-1} + x_{t-1} - x_{t-1} + \beta_0 x_{t-1} - \beta_0 x_{t-1} + \alpha_1 x_{t-1} - \alpha_1 x_{t-1}$

• rearranging terms, we obtain the error correction model:

$$\Delta y_t = (\alpha_1 - 1)(\underbrace{y_{t-1} - \frac{\alpha_0}{1 - \alpha_1} - \frac{\beta_0 + \beta_1}{1 - \alpha_1}x_{t-1}}_{ECT: \ \epsilon_{t-1}^-}) + \beta_0 \Delta x_t + \varepsilon_t$$

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Stationarity and nonstationarity o oocoo oocooo	Testing for integration 0 0000 00000	Cointegration 0 000	Error correction model ○ ●○○○
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$$\Delta y_t = (\alpha_1 - 1)(\underbrace{y_{t-1} - \frac{\alpha_0}{1 - \alpha_1} - \frac{\beta_0 + \beta_1}{1 - \alpha_1}x_{t-1}}_{ECT: \ \epsilon_{t-1}^-}) + \beta_0 \Delta x_t + \varepsilon_t$$

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		Cointegration	Error correction model
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Error correction model (2)

Procedure:

- estimate the cointegrating relationship
- estimate the ECM, using differenced variables and lagged residuals from the cointegrating relationship
- variables are cointegrated when $(\alpha_1 1) < 0$
 - > 0: disequilibrium expands
 - = 0: no error correction
 - (-1; 0): error correction (close to 0: slow, close to -1: quick; $half - life = \frac{\ln(0.5)}{\ln(\alpha_1)}$)
 - = -1: full error correction in 1 period
 - ullet < -1: overshooting, oscillatory adjustment

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Error correction model (2)

Procedure:

- estimate the cointegrating relationship
- estimate the ECM, using differenced variables and lagged residuals from the cointegrating relationship
 - variables are cointegrated when $(lpha_1-1)<0$
 - > 0: disequilibrium expands
 - = 0: no error correction
 - (-1;0): error correction (close to 0: slow, close to -1: quick; $half - life = \frac{\ln(0.5)}{\ln(\alpha_1)}$)
 - = -1: full error correction in 1 period
 - ullet < -1: overshooting, oscillatory adjustment

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Error correction model (2)

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ECM			



Consider the following variables:

- log real wages (dependent)
- log labour productivity
- unemployment

What is their order of integration?

Are they cointegrated?

Are the signs in the cointegrating relationship economically reasonable?

Is there error correction mechanism at work?

What is the half-life of the adjustment?

Example inspired by Zasova, Melihovs (2005)

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ECM			



• Greene:

- Chapter 20 Models With Lagged Variables
- Chapter 21 Time-Series Models
- Chapter 22 Nonstationary Data

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(9) Nonstationarity. ECM

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