Testing for heteroskedasticity 0 00 0

Dealing with heteroskedasticity 0 00000000

Lecture 7: Heteroskedasticity

Econometric Methods

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(7) Heteroskedasticity

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Outline



What is heteroskedasticity?

- 2 Testing for heteroskedasticity
 - White
 - Goldfeld-Quandt
 - Breusch-Pagan
- Obealing with heteroskedasticity
 - Robust standard errors
 - Weighted Least Squares estimator

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What is heteroskedasticity? Testing for h	eteroskedasticity Dealing with heteroskedasticity
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Outline



- 2 Testing for heteroskedasticity
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What is	heteroskedasticity?
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Theoretical background

Recall: variance-covariance matrix of ${m arepsilon}$

 $E\left(\varepsilon\varepsilon^{T}\right) =$ $= \begin{bmatrix} \operatorname{var}(\varepsilon_1) & \operatorname{cov}(\varepsilon_1\varepsilon_2) & \dots & \operatorname{cov}(\varepsilon_1\varepsilon_T) \\ \operatorname{cov}(\varepsilon_1\varepsilon_2) & \operatorname{var}(\varepsilon_2) & \dots & \operatorname{cov}(\varepsilon_2\varepsilon_T) \\ \vdots & \vdots & \ddots & \vdots \\ \operatorname{cov}(\varepsilon_1\varepsilon_T) & \operatorname{cov}(\varepsilon_2\varepsilon_T) & \dots & \operatorname{var}(\varepsilon_T) \end{bmatrix} =$ $OLS \text{ assumptions} \begin{bmatrix} \sigma^2 & 0 & \dots & 0 \\ 0 & \sigma^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma^2 \end{bmatrix}$

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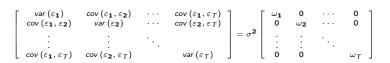
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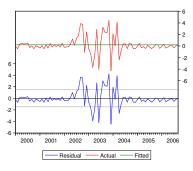
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Theoretical background

Heteroskedasticity





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What	is heteroskedastic	ity?
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Theoretical background

Non-spherical disturbances

, v	variance-covariance	serial corr	elation
	matrix		
	of the error term	absent	present
skedasticity	absent	$\left[\begin{array}{cccccc} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & \sigma^2 \end{array}\right]$	$\sigma^{2} \begin{bmatrix} 1 & \omega_{12} & \cdots & \omega_{17} \\ \omega_{12} & 1 & \cdots & \omega_{27} \\ \vdots & \vdots & \ddots \\ \vdots & \vdots & \ddots \\ \omega_{17} & \omega_{27} & 1 \end{bmatrix}$
hetero-	present	$\sigma^{2} \begin{bmatrix} \omega_{1} & 0 & \cdots & 0 \\ 0 & \omega_{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & \omega_{T} \end{bmatrix}$	Ω (???)

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Theoretical background

Consequences of heteroskedasticity

Common features of non-spherical disturbances (see serial correlation):

- no bias, no inconsistency...
- ...but inefficiency!

of OLS estimates.

Unlike serial correlation...

heteroskedasticity can occur both in

- time series data (e.g. high- and low-volatility periods in financial markets)

- **cross-section data** (e.g. variance of disturbances depends on unit size or some key explanatory variables)

What is heteroskedasticity? ○ ○○○○●	Dealing with heteroskedasticity 0 00000000
Theoretical background	

Exercise (1/3)

Credit cards

Based on client-level data, we fit a model that that explains the credit-card-settled expenditures with:

- age;
- income;
- squared income;
- house ownership dummy.

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White

White test (1)

- Step 1: **OLS regression** $y_i = \mathbf{x}_i \boldsymbol{\beta} + \varepsilon_i$ $\hat{\boldsymbol{\beta}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$ $\hat{\varepsilon}_i = y_i - \mathbf{x}_i \hat{\boldsymbol{\beta}}$
- Step 2: auxiliary regression equation $\hat{\varepsilon}_i^2 = \sum_{k,l} x_{k,i} x_{l,i} \beta_{k,l} + v_i$

E.g. in a model with a constant and 3 regressors x_{1t} , x_{2t} , x_{3t} , the auxiliary model contains the following explanatory variables:

 $\underbrace{\text{constant}, \ X_{1t}, X_{2t}, X_{3t}, X_{1t}, X_{2t}, X_{3t}, \underbrace{X_{1t} \cdot X_{2t}, X_{2t} \cdot X_{3t}, X_{1t} \cdot X_{3t}}_{}_{}}_{}$

cross terms

• IDEA: Without heteroskedasticity, the R^2 of the auxiliary equation should be low.

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White

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- Step 2: auxiliary regression equation $\hat{\varepsilon}_{i}^{2} = \sum_{k,l} x_{k,i} x_{l,i} \beta_{k,l} + v_{i}$

E.g. in a model with a constant and 3 regressors $\mathbf{x}_{1t}, \mathbf{x}_{2t}, \mathbf{x}_{3t}$, the auxiliary model contains the following explanatory variables: constant, $x_{1t}, x_{2t}, x_{3t}, x_{1t}^2, x_{2t}^2, x_{3t}^2, x_{1t} \cdot x_{2t}, x_{2t} \cdot x_{3t}, x_{1t} \cdot x_{3t}$.

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cross terms

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White		

White test (2)

White test

 H_0 : heteroskedasticity absent

H1 : heteroskedasticity present

$$W = TR^2 \sim \chi^2 \left(k^* \right)$$

 k^* – number of explanatory variables in the auxiliary regresson (excluding constant)

CAUTION! It's a weak test (i.e. low power to reject the null)

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Goldfeld-Quandt

Goldfeld-Quandt test

- split the sample (T observations) into 2 subsamples ($T = n_1 + n_2$)
- test for equality of error term variance in both subsamples

Goldfeld-Quandt

 $H_0: \sigma_1^2 = \sigma_2^2$ equal variance in both subsamples (homoskedasticity) $H_1: \sigma_1^2 > \sigma_2^2$ higher variance in the subsample indexed as 1

$$F(n_1 - k, n_2 - k) = \frac{\sum_{i=1}^{n_1} \hat{\varepsilon}_i^2 / (n_1 - k)}{\sum_{i=n_1+1}^{T} \hat{\varepsilon}_i^2 / (n_2 - k)}$$

CAUTION! This makes sense only when we index the subsample with higher variance as 1. Otherwise we never reject H_0 .

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Breusch-Pagan

Breusch-Pagan test

 variance of the disturbances can be explained with a variable set contained in matrix Z (like explanatory variables for y in the matrix X)

Breusch-Pagan test

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Breusch-Pagan

Exercise (2/3)

Credit cards

- Does the White test detect heteroskedasticity?
- Split the sample into two equal subsamples: high-income and low-income. Check if the variance differs between the two sub-samples. (You need to sort the data and restrict the sample to a sub-sample twice, each time calculating the appropriate statistics.)
- Perform the Breusch-Pagan test, assuming that the variance depends only on the income and squared income (and a constant).

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Robust standard errors	

White robust SE

- unlike Newey-West robust SE (robust to both serial correlation and heteroskedasticity), White's SE robust only to heteroskedasticity (the former were proposed later and generalized White's work)
- White (1980):

$$Var\left(\hat{\boldsymbol{\beta}}\right) = \left(\boldsymbol{X}^{T}\boldsymbol{X}\right)^{-1} \left(\sum_{t=1}^{T} \hat{\varepsilon}_{t}^{2} \mathbf{x}_{t} \mathbf{x}_{t}^{T}\right) \left(\boldsymbol{X}^{T}\boldsymbol{X}\right)^{-1}$$

• they share the same features as Newey-West SE (see: serial correlation), i.e. correct statistical inference without improving estimation efficiency of the parameters themselves

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Weighted Least Squares estimator (1)

•
$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$
 $\varepsilon \sim \left(E\left[\boldsymbol{\varepsilon} \right] = \mathbf{0}, E\left[\boldsymbol{\varepsilon} \boldsymbol{\varepsilon}^{T} \right] = \mathbf{\Omega} \right)$

• Recall the GLS estimator. Under heteroskedasticity, we know that

the variance-covariance matrix
$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & \omega_2 & \dots & 0 \\ 0 & \omega_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \omega_T \end{bmatrix}$$
, which implies $\boldsymbol{\Omega}^{-1} = \begin{bmatrix} \frac{1}{\omega_1} & 0 & \dots & 0 \\ 0 & \frac{1}{\omega_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{\omega_T} \end{bmatrix}$.

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Weighted Least Squares estimator (2)

• Knowing the vector
$$\begin{bmatrix} \frac{1}{\omega_{1}} & \frac{1}{\omega_{2}} & \cdots & \frac{1}{\omega_{T}} \end{bmatrix}$$
 we can immediately apply GLS:
 $\hat{\beta}^{WLS} = \begin{bmatrix} \mathbf{X}^{T} \mathbf{\Omega}^{-1} \mathbf{X} \end{bmatrix}^{-1} \mathbf{X}^{T} \mathbf{\Omega}^{-1} \mathbf{y} =$

$$= \begin{pmatrix} \begin{bmatrix} \mathbf{x}_{1}^{T} & \mathbf{x}_{2}^{T} & \cdots & \mathbf{x}_{T}^{T} \end{bmatrix} \begin{bmatrix} \frac{1}{\omega_{1}} & 0 & \cdots & 0 \\ 0 & \frac{1}{\omega_{2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\omega_{T}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{1} \\ \mathbf{x}_{2} \\ \vdots \\ \mathbf{x}_{T} \end{bmatrix} \end{pmatrix}^{-1} \cdot$$

$$\cdot \begin{bmatrix} \mathbf{x}_{1}^{T} & \mathbf{x}_{2}^{T} & \cdots & \mathbf{x}_{T}^{T} \end{bmatrix} \begin{bmatrix} \frac{1}{\omega_{1}} & 0 & \cdots & 0 \\ 0 & \frac{1}{\omega_{2}} & \cdots & 0 \\ 0 & \frac{1}{\omega_{2}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\omega_{T}} \end{bmatrix} \begin{bmatrix} \mathbf{y}_{1} \\ \mathbf{y}_{2} \\ \vdots \\ \mathbf{y}_{T} \end{bmatrix} =$$

$$= \begin{pmatrix} \sum_{i=1}^{T} \frac{1}{\omega_{i}} \mathbf{x}_{i}^{T} \mathbf{x}_{i} \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^{T} \frac{1}{\omega_{i}} \mathbf{x}_{i}^{T} \mathbf{y}_{i} \end{pmatrix}$$

 This vector can hence be interpreted as a vector of weights, associated with individual observations in the estimation (hence: "weighted" least squares).

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Testing for heteroskedasticity 0 00 0 Weighted Least Squares estimator

$$\hat{\boldsymbol{\beta}}^{WLS} = \left(\sum_{i=1}^{T} \frac{1}{\omega_i} \boldsymbol{x}_i^T \boldsymbol{x}_i\right)^{-1} \left(\sum_{i=1}^{n} \frac{1}{\omega_i} \boldsymbol{x}_i^T \boldsymbol{y}_i\right) = \\ = \left(\sum_{i=1}^{n} \frac{\mathbf{x}_i}{\sqrt{\omega_i}} \frac{\mathbf{x}_i^T}{\sqrt{\omega_i}}\right)^{-1} \left(\sum_{i=1}^{T} \frac{\mathbf{x}_i^T}{\sqrt{\omega_i}} \frac{\mathbf{y}_i}{\sqrt{\omega_i}}\right)$$

• The WLS estimation is hence equivalent to OLS estimation using data transformed in the following way:

$$\mathbf{y}^* = \begin{bmatrix} \begin{array}{c} y_1/\sqrt{\omega_1} \\ y_2/\sqrt{\omega_2} \\ \vdots \\ y_T/\sqrt{\omega_T} \end{array} \end{bmatrix} \qquad \mathbf{X}^* = \begin{bmatrix} \begin{array}{c} \mathbf{x}_1/\sqrt{\omega_1} \\ \mathbf{x}_2/\sqrt{\omega_2} \\ \vdots \\ \mathbf{x}_T/\sqrt{\omega_T} \end{bmatrix}$$

Conclusion

Weights for individual observations are the inverse of the disturbance variance in individual periods. Under OLS, these weights are a unit vector.

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Testing for heteroskedasticity 0 00 0 Weighted Least Squares estimator

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Weighted Least Squares estimator

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Weighted Least Squares estimator	

Unknown and, with T observations, cannot be estimated. The most popular solutions include:

- Way 1:
 - Split the sample into subsamples.
 - Estimate the model in each subsample via OLS to obtain the vector $\hat{\varepsilon}$.
 - In every subsample *i* estimate the variance of error terms $\hat{\sigma}_i^2$.
 - Assign the weight $\frac{1}{\hat{\sigma}_i^2}$ to all the observations in the subsample *i*.

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• Way 2:

• Estimate the model via OLS to obtain the vector $\hat{\varepsilon}$.

- Regress $\hat{\varepsilon}_t^2$ against a set of potential explanatory variables (when done automatically, usually all the regressors from the base equation plus possibly their squares).
- Take theoretical value of $\hat{\varepsilon}_t^2$ from this regression say, e_t^2 as a proxy of variance (one cannot use $\hat{\varepsilon}_t^2$ itself, as it does not measure variance adequately – it is just one draw from a distribution, while the theoretical value summarizes a number of draws made under similar conditions regarding the explanatory variables for variance).
- Use $\frac{1}{e^2}$ as weights for individual observations t.
- In practice, it is common to regress $ln(\hat{\varepsilon}_t^2)$ rather than $\hat{\varepsilon}_t^2$. In this way we compute $(\hat{\varepsilon}_t^2) = exp\left(ln\left(\hat{\varepsilon}_t^2\right)\right)$ which blocks negative values of error terms' variance.

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	Dealing with heteroskedasticity ° 0000∙000

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 - Regress $\hat{\varepsilon}_t^2$ against a set of potential explanatory variables (when done automatically, usually all the regressors from the base equation plus possibly their squares).
 - Take theoretical value of $\hat{\varepsilon}_{t}^{2}$ from this regression say, e_{t}^{2} as a proxy of variance (one cannot use $\hat{\varepsilon}_{t}^{2}$ itself, as it does not measure variance adequately – it is just one draw from a distribution, while the theoretical value summarizes a number of draws made under similar conditions regarding the explanatory variables for variance).
 - Use $\frac{1}{e^2}$ as weights for individual observations t.
 - In practice, it is common to regress $ln(\hat{\varepsilon}_t^2)$ rather than $\hat{\varepsilon}_t^2$. In this way we compute $(\hat{\varepsilon}_t^2) = exp\left(ln\left(\hat{\varepsilon}_t^2\right)\right)$ which blocks negative values of error terms' variance.

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	Dealing with heteroskedasticity S 0000●000

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Weighted Least Squares estimator	

Exercise (3/3)

Credit cards

- Split the sample into two equal subsamples and use WLS (way 1).
- Output Set of the explanatory variables and their sqares as the regressors in the variance equation and use WLS (way 2).
- Compare the parameter values between OLS and the two variants of WLS;
- Compare variable significance between OLS, OLS with White's robust standard errors and the two variants of WLS.

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Weighted Least Squares estimator	



• Greene: chapter "Generalized Regression Model and Heteroscedasticity"

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Weighted Least Squares estimator	

Homework

Gasoline demand model

Verify the presence of heteroskedasticity in the model considered in the previous lecture.

Programming

Write an R-function performing an automated heteroskedasticity correction a la "way 2" in this presentation, using WLS, and using all the right-hand side variables as potential determinants of error term variance.

(7) Heteroskedasticity

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